Atmosphere

1 Overview

The performance of aircraft, spacecraft and their engines depend on atmosphere in which they operate, primarily **density and viscosity**. Density and viscosity are functions of **altitude**:

1. Density (ρ) varies with pressure (p) and temperature (T)
2. Viscosity (μ) varies only with temperature (T)

2 Standard Atmosphere

The standard atmosphere is defined from **Perfect Gas Law:**

$$p = \rho R T \tag{1}$$

There are four units of temperature:

1. Celsius (^oC)
2. Fahrenheit (^oF)
3. Rankine (R)
4. Kelvin (K)

And their conversions are listed below:

$$^oF = (^oC \times \frac{9}{5}) + 32 \tag{2}$$

$$R = (^oF) + 459.7 \tag{3}$$

$$K = (^oC) + 273.15 \tag{4}$$

To simplify the problem, the atmosphere can be regarded as **homogeneous** gas of **uniform composition** that satisfies the perfect gas law. Established by International Civil Aviation Organization (ICAO), the **standard sea level properties**:

$$g_0 = 32.17\ ft/s^2 = 9.806\ m/s^2 \tag{5}$$

$$p_0 = 2116.2\ lb/ft^2 = 1.013 \times 10^5\ N/m^2 \tag{6}$$

$$T_0 = 59\,^oF = 518.7\,R = 15^oC = 288.2\,K \tag{7}$$

$$\rho_0 = 0.002377\ slugs/ft^3 = 1.225\,kg/m^3 \tag{8}$$

3 Variation with Altitude

3.1 Regions of the Atmosphere

Normally atmosphere could be divided into four regions (from low to high):

1. Troposphere
2. Stratosphere
3. Ionosphere
4. Exosphere-rarefield

The characteristics of each region are shown below:

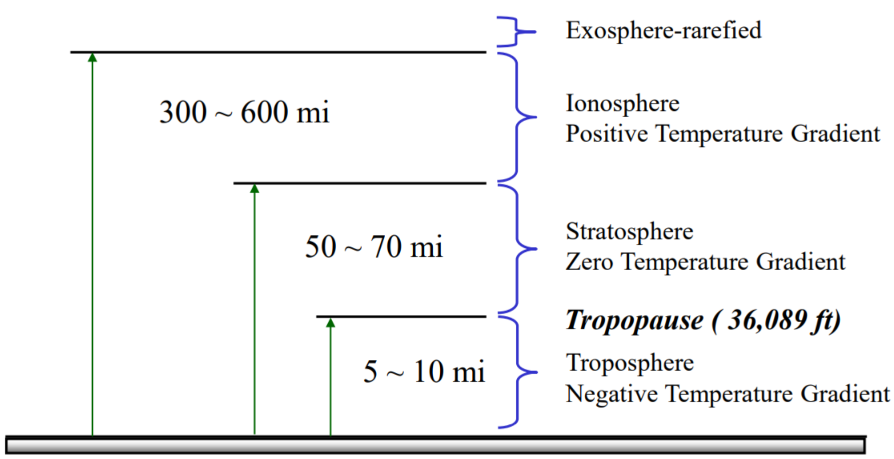

Figure 1: Regions of Atmosphere

For aircraft, only **troposphere** and **stratosphere** are important.

3.2 Variations under Tropopause

Below Tropopause, we assume there is a **constant drop** of temperature from **sea level** to altitude:

$$T = T_1 + a(h - h_1) \tag{9}$$

Here a is the **lapse rate**, with the unit of $^oF/ft$:

$$a = -0.00356616 \tag{10}$$

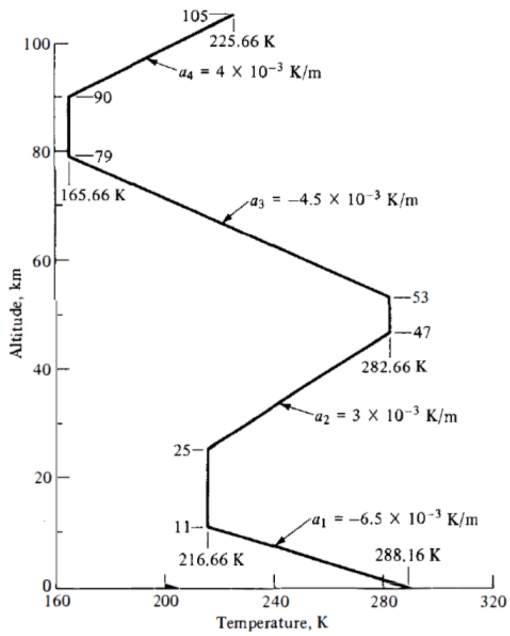

Figure 2: Temperature Variation

T_1 and h_1 are the reference values. At sea level, we have $T_1 = T_0$ and $h_1 = 0$. **Below Tropopause**, we have the following relations:

$$\frac{T}{T_0} = \theta = 1 + \frac{a}{T_0}h = 1 - 6.875 \times 10^{-6}h \tag{11}$$

$$\frac{p}{p_0} = \delta = \theta^{\frac{g}{aR}} = \theta^{5.2561} \tag{12}$$

$$\frac{\rho}{\rho_0} = \sigma = \theta^{\frac{g}{aR}-1} = \theta^{4.2561} \tag{13}$$

3.3 Variations above Tropopause

Above tropopause, the standard temperature is assumed **constant** and equal to $-69.7\ ^oF$. Then, we have the following relations (relative to standard sea level values):

$$\frac{p}{p_0} = 0.2234 \exp\left[-\frac{h - 36089}{20806.7}\right] \tag{14}$$

$$\frac{\rho}{\rho_0} = 0.2971 \exp\left[-\frac{h - 36089}{20806.7}\right] \tag{15}$$

4 Types of Airspeed

4.1 Indicated Airspeed (IAS)

IAS is the direct reading from the **airspeed indicator**. This represents the aircraft's speed **through the air**, may not be its speed **across the ground**. The system uses the difference between the total pressure (measured by the pitot probe) and the static pressure (measured by the static ports) to determine the dynamic pressure which is converted to an airspeed reading.

Recall the total (stagnation) pressure definition (incompressible flow):

$$P_{static} + P_{dynamic} = P_{total} \tag{16}$$

And the dynamic pressure definition:

$$P_{dynamic} = \rho g h + \frac{1}{2}\rho V^2 \tag{17}$$

Assume the change in height along the streamline is negligible, so $\rho g h$ could be neglected. Therefore, the airspeed could be calculated as:

$$IAS = \sqrt{\frac{2(P_{total} - P_{static})}{\rho_0}} \tag{18}$$

Notice here we use ρ_0, because the airspeed indicator in the cockpit is always calibrated to **sea level density on a standard day**.

4.2 Calibrated Airspeed (CAS)

CAS is the indicated airspeed corrected for **instrument position and instrument error**. This is a function of each unique and **the position of its pitot tube**. There is no direct reading of CAS! The pilot must refer to the operating handbook for a table for that particular aircraft.

4.3 Equivalent Airspeed (EAS)

EAS is defined as the speed at sea level, under ISA (International Standard Atmosphere) conditions, that would produce the same incompressible dynamic pressure that is produced at the true airspeed and the altitude at which the vehicle is flying. It is this definition that makes EAS a useful airspeed measurement for aeronautical engineers as it provides a convenient way to calculate loading on the airframe, or **handling qualities as the dynamic pressure provided is an equivalent sea level pressure without the need to correct for altitude or temperature.**

Indicated and Calibrated airspeed is based on the formulation of Bernoulli's equation, which assumes that the fluid (air in this case) is incompressible. Bernoulli's experiments were performed in water where this assumption is valid, but compressibility effects in air start to become significant at Mach numbers above 0.3. **Divergence between CAS and EAS will be seen at speeds above 200 kts and altitudes above 10 000 ft.** CAS must therefore be corrected for compressibility effects to determine

EAS as an intermediate step to calculate the True Airspeed (TAS). In compressible flow (subsonic regime):

$$P_{static} + q_c = P_{stagnation} \tag{19}$$

Compressibility effects can be accounted for through the calculation of the impact pressure, which is a function of the Mach number. Notice the following formulas only work for subsonic regime, in supersonic regime the shock equation should be included in impact pressure calculation.

$$q_c = P_{static}\left[\left(1 + \frac{\gamma - 1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}} - 1\right] \tag{20}$$

$$M = \sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{q_c}{P_{static}} + 1\right)^{\frac{\gamma-1}{\gamma}} - 1\right]} \tag{21}$$

$$CAS = a_0\sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{q_c}{P_{static,0}} + 1\right)^{\frac{\gamma-1}{\gamma}} - 1\right]} \tag{22}$$

$$EAS = a_0 M\sqrt{\frac{P_{static}}{P_{static,0}}} \tag{23}$$

Here:

- q_c: Impact pressure
- M: Mach number
- P_{static}: Static pressure
- $P_{static,0}$: Static sea level pressure (ISA)
- a_0: Sonic Speed at sea level ISA (661.47 knots, 340.29 m/s)

Another way to transfer Mach number to EAS is to transfer Mach to TAS first, which is mentioned below.

4.4 True Airspeed (TAS)

Because an airspeed indicator is **calibrated for standard sea level conditions**, when the airplane is flying at altitude, the airspeed is not correctly reflected. The amount of error is a function of temperature and altitude. **The true airspeed is the speed that the aircraft travels relative to the air mass in which it is flying.** The true airspeed is equal to the ground speed in cases where there is no wind, and is used mostly for flight planning and when quoting aircraft performance specifications.

Recall the VN diagram:

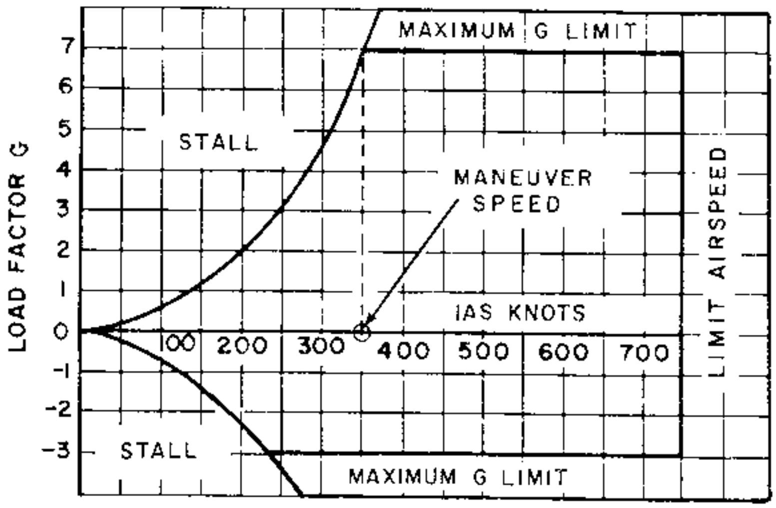

Figure 3: VN Diagram

The curve at the left hand side is caused by the stall limit. Recall the lift equation:

$$L = \frac{1}{2}\rho_\infty V_\infty^2 S C_L \tag{24}$$

At the stall limit:

$$nW = \frac{1}{2}\rho_\infty V_\infty^2 S C_{L_{max}} \tag{25}$$

$$n \propto \rho_\infty V_\infty^2 \tag{26}$$

Now we want to get rid of the variation of density due to altitude change:

$$\frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}\rho_{SL} V_E^2 \tag{27}$$

$$TAS = EAS\sqrt{\frac{\rho_0}{\rho}} \tag{28}$$

Here ρ is the actual air density, and ρ_0 is the sea level air density. It could also be calculated using Mach number:

$$TAS = Ma \tag{29}$$

$$a = \sqrt{\gamma R T} \tag{30}$$

Here a is the sonic speed at given outside air temperature.

5 Other Terminology

- Δ_{ISA}: is a term used in aerospace to indicate the deviation of the actual atmospheric temperature from the standard temperature defined by the International Standard Atmosphere (ISA) at a given altitude. In formula:

$$\Delta_{ISA} = T_{OA} - T_{ISA} \tag{31}$$