Equations of Motion

1 Four Forces of Flight

1.1 Definitions

The definitions of lift (L), drag (D), weight (W) and thrust (T) are shown below:

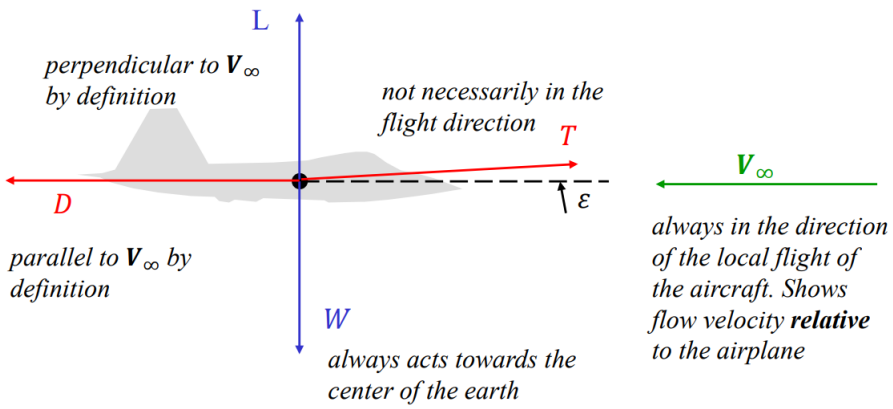

Figure 1: Four Forces Definitions

Here ε is the **angle between thrust direction and flight path**, but it is not **angle of attack (AoA, α)**. Based on the definition of AoA, it is the angle between the **chord line of an airfoil and the oncoming airflow.**

1.2 Climbing Flight

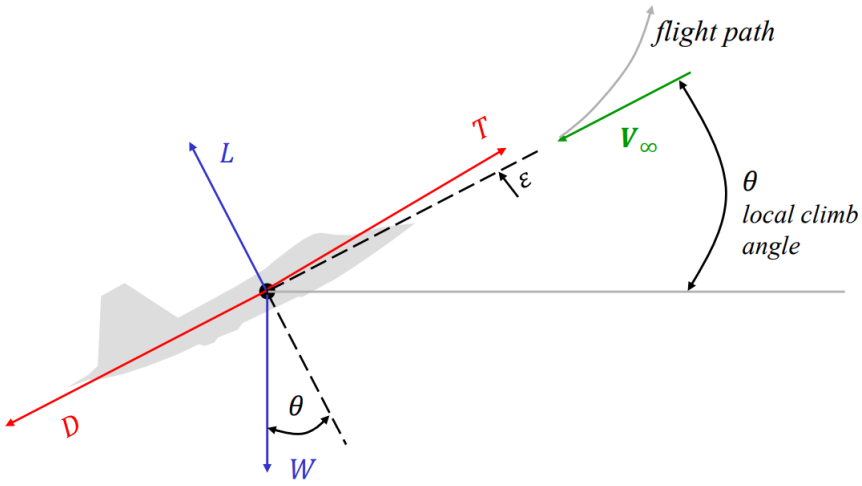

Figure 2: Climbing Flight

1.3 Turning/Banking Flight

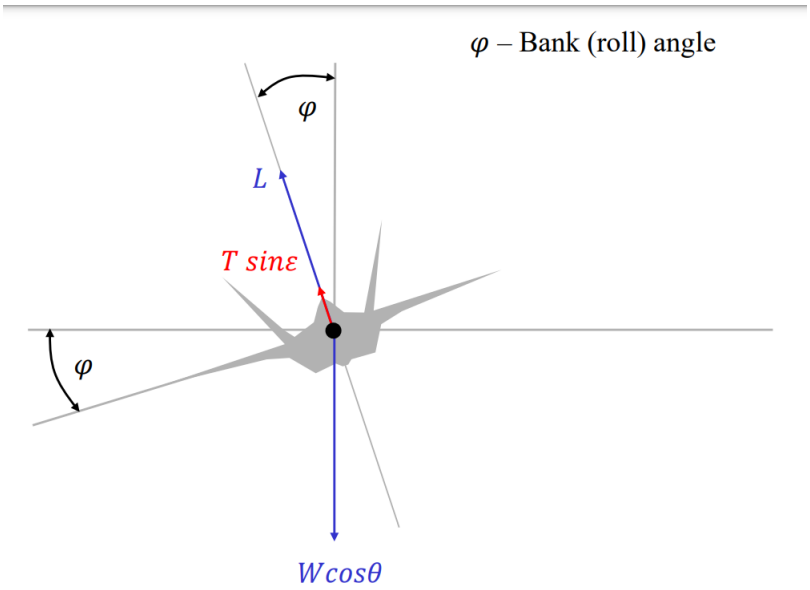

Figure 3: Banking Flight

1.4 Center of Gravity Simplification

Replace AC with point mass as its center of gravity, because we only care about translational motion:

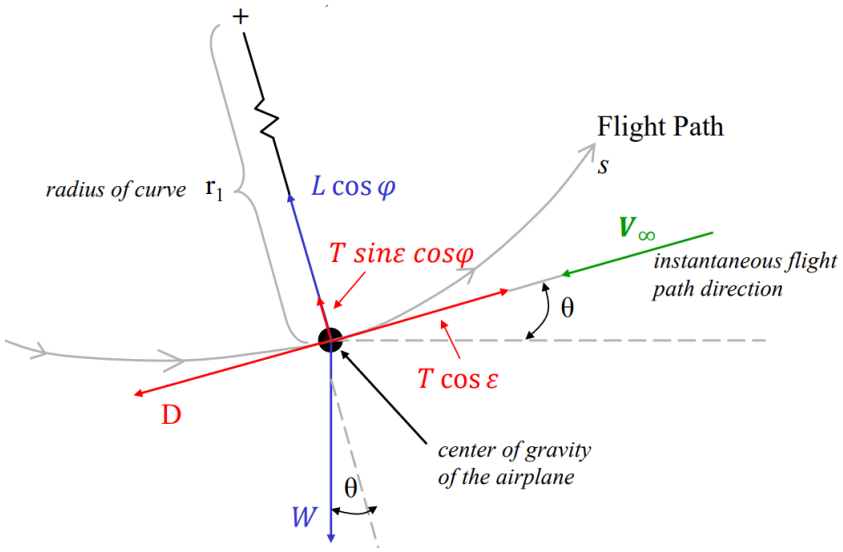

Figure 4: Banking Flight

2 First Equation of Motion

For the direction **parallel to the flight path**, we have the force:

$$F_{||} = T\cos\varepsilon - D - W\sin\theta \tag{1}$$

And the acceleration:

$$a_{||} = \frac{dU_\infty}{dt} \tag{2}$$

So we have the **first equation of motion:**

$$\boxed{m\frac{dU_\infty}{dt} = T\cos\varepsilon - D - W\sin\theta} \tag{3}$$

3 Second Equation of Motion

For the direction **perpendicular to the flight path**, we have the force:

$$F_\perp = L\cos\varphi + T\sin\varepsilon\cos\varphi - W\cos\theta \tag{4}$$

And the **radial acceleration:**

$$a_\perp = \frac{U_\infty^2}{r_1} \tag{5}$$

Therefore, the **second equation of motion** is:

$$\boxed{m\frac{U_\infty^2}{r_1} = L\cos\varphi + T\sin\varepsilon\cos\varphi - W\cos\theta} \tag{6}$$

4 Third Equation of Motion

If we look at flight path from a top view:

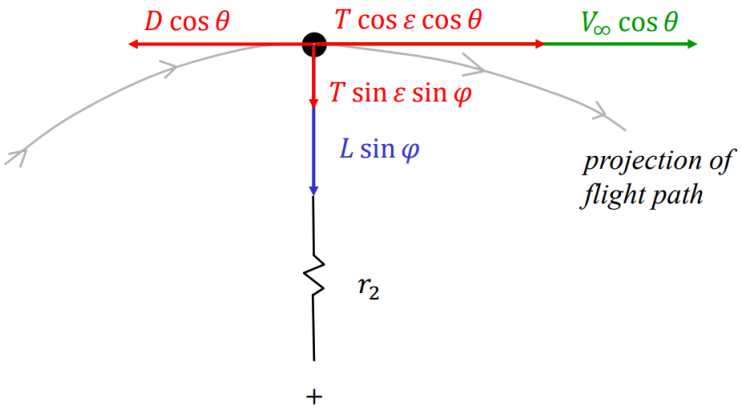

Figure 5: Top View on Horizontal Plane

At the direction **perpendicular to the flight path in the horizontal plane**, the force is:

$$F_2 = L\sin\varphi + T\sin\varepsilon\sin\varphi \tag{7}$$

The radial acceleration is:

$$a_2 = \frac{(U_\infty\cos\theta)^2}{r_2} \tag{8}$$

Finally we have the **third equation of motion:**

$$\boxed{m\frac{(U_\infty\cos\theta)^2}{r_2} = L\sin\varphi + T\sin\varepsilon\sin\varphi} \tag{9}$$

5 Summary

1. These three equations describe the **translational motion** of an aircraft through 3D space over a **flat earth**.

2. There are 3 additional equations of motion that describes **rotational** motion of the aircraft.

3. Do not assume a yaw component. The free stream velocity vector is assumed always parallel to the symmetry plane of the aircraft.