Lift, Drag and Moments

1 Definition

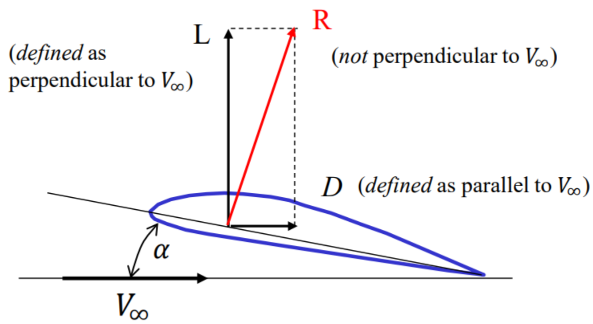

Figure 1: Lift, Drag

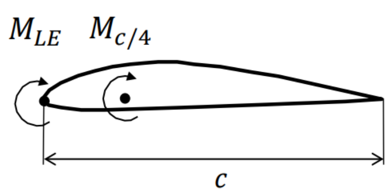

Figure 2: Moment

Aerodynamic Lift: the force that enables an aircraft (or any other object) to rise against gravity due to the motion and shape of the object in a fluid, usually air for aircraft.

Aerodynamic Drag: the force that opposes the motion of an object through a fluid.

Aerodynamic Moment: a moment refers to a **rotational force or torque** applied to a body, typically an aircraft. Just as forces (like lift, weight, thrust, and drag) can cause linear motion, moments can cause rotational motion. For an aircraft, these rotations usually occur about the aircraft's principal axes: pitch, roll, and yaw. By convention, a moment which rotates a body causing an **increase in angle of attack is positive.**

2 Reference Area

S is the reference area used to calculate the aerodynamic coefficients. But what exactly it is?

1. **S as wetted area:** the surface upon which the pressure and shear distribution act. **Not common.**

2. **S as planform area:** the projected area we see when **looking down the wing or aircraft**. **Most common.**

3. **S as base area:** used when analyzing slender bodies, such as missiles.

For the planform area, we have:

$$S = b \times MAC \tag{1}$$

where b is the **wing span** (the distance from one wingtip to the other) and MAC is the **mean aerodynamic chord**, which is the average chord length account the varying chord length across the span.

3 Lift and Drag Calculation

To perform analysis of momentum conservation, we need to define the control surface. Define CS based on edge streamlines, and assume that the streamlines being considered are sufficiently far away that $p = p_\infty$:

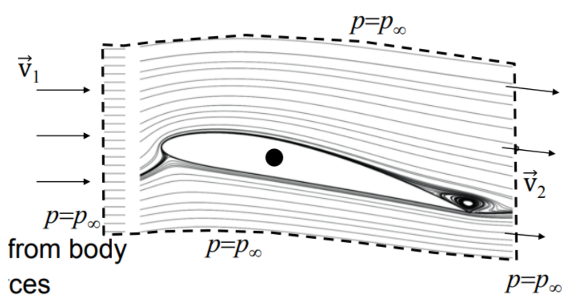

Figure 3: Control Surface

Now we have the assumptions:

1. Inviscid flow far from body

2. Neglect body forces

3. Steady, stationary CS

Recall the NS Equations:

$$\underline{\boldsymbol{F}} - \int_{CS} p\underline{\boldsymbol{n}}dA + \cancelto{0}{\int_{CS} \underline{\boldsymbol{\tau}}dA} + \cancelto{0}{\int_{CV} \rho\underline{\boldsymbol{f}}dV} = \cancelto{0}{\frac{d}{dt}\int_{CV} \rho\underline{\boldsymbol{u}}dV} + \int_{CS} \rho\underline{\boldsymbol{u}}(\underline{\boldsymbol{un}})dA \tag{2}$$

Here $\underline{\boldsymbol{F}}$ represents the total force acting on CS by solid. Under our assumptions, pressure is same on all parts of CS, so integral is zero:

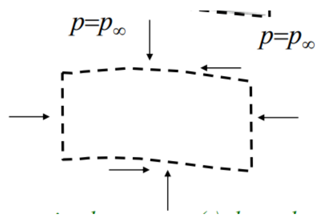

Figure 4: Pressure Integration

Finally we have:

$$\boxed{\underline{\boldsymbol{F}}_{\text{solid on CS}} = \int_{CS} \rho\underline{\boldsymbol{u}}(\underline{\boldsymbol{un}})dA} \tag{3}$$

Now, we define the x, y directions as shown below:

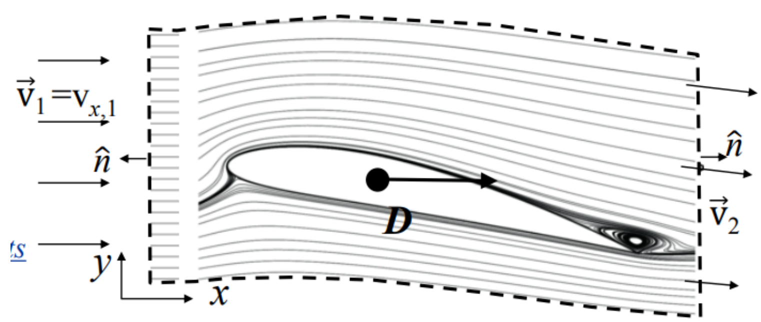

Figure 5: X, Y directions Definitions

3.1 Drag

Recall that the definition of drag, it is the force **fluid in CS exerts on the solid**, so we need to reverse the direction. Therefore in x-direction:

$$-\underline{\boldsymbol{D}} = \int_{CS} \rho u_x(\underline{\boldsymbol{un}})dA \tag{4}$$

Based on the direction of the normal vector, we can have:

$$-D = \int_{RHS} \rho u_{x,2}u_{x,2}dA - \int_{LHS} \rho u_{x,1}u_{x,1}dA \tag{5}$$

Recall the continuity equation:

$$\int_{LHS} \rho u_{x,1} dA = \int_{RHS} \rho u_{x,2} dA \tag{6}$$

Therefore we can get:

$$D = \int_{RHS} \rho u_{x,2}(u_{x,1} - u_{x,2}) dA \tag{7}$$

Define a per span basis:

$$dA = s dy \tag{8}$$

And define $u_1 = u_\infty$:

$$\boxed{\frac{D}{s} = \int_{RHS} \rho u_{x,2}(u_\infty - u_{x,2}) dy} \tag{9}$$

As a conclusion, drag is associated with **velocity deficit (wake) behind the airfoil**, which is a drop in **x-direction velocity**.

Finally, we define the **drag coefficient** as:

$$\boxed{C_D = \frac{D}{\frac{1}{2}\rho_\infty u_\infty^2 S}} \tag{10}$$

3.2 Lift

Also for the y direction:

$$-\frac{L}{s} = \int_{RHS} \rho u_{y,2} u_{x,2} dy - \int_{LHS} \rho u_{y,1} u_{x,1} dy \tag{11}$$

Recall that we assume the initial flow is only in x-direction, so $u_{y,1} = 0$. Then:

$$\boxed{\frac{L}{s} = -\int_{RHS} \rho u_{x,2} u_{y,2} dy} \tag{12}$$

In conclusion, airfoil produces lift by **giving flow downward momentum**. Lift is associated with a downward velocity in the wake of the airfoil.

Finally, we define the **lift coefficient** as:

$$\boxed{C_L = \frac{L}{\frac{1}{2}\rho_\infty u_\infty^2 S}} \tag{13}$$

4 Center of Pressure vs Aerodynamic Center

4.1 Center of Pressure

The center of pressure is the point where the resultant force **due to the pressure field** acts on the object. The magnitude of the resultant force is the value of the **integral of the pressure field.**

For an airfoil moving through a fluid, the center of pressure is the point where the pressure field can be reduced to a **force with no associated moment.**

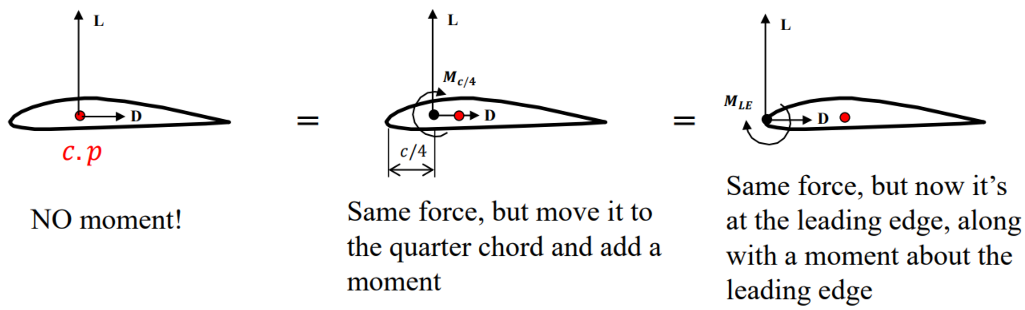

Figure 6: Center of Pressure

4.2 Aerodynamic Center

The aerodynamic center (x_{ac}) is the point on an airfoil about which the aerodynamic moment is **independent of the lift coefficient**, or in other words **angle of attack**:

$$\frac{dc_m}{dc_l}|_{x_{ac}} = 0 \tag{14}$$

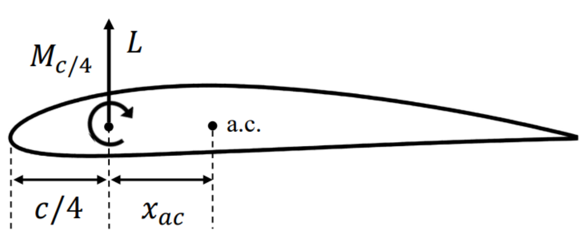

Figure 7: Aerodynamic Center

4.3 Comparison

1. The center of pressure changes with **angle of attack**, as pressure distribution changes shape.

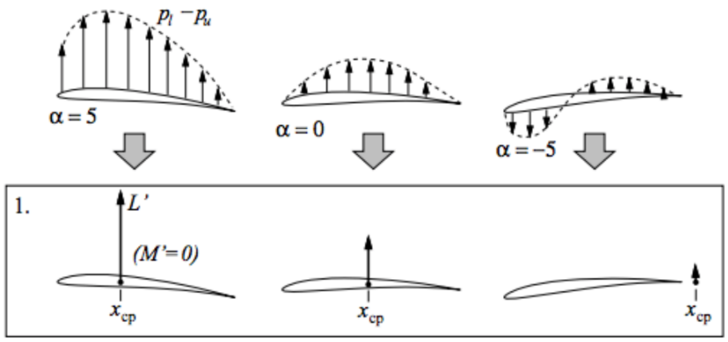

Figure 8: Center of Pressure not a Fixed Point

2. The changing location of the center of pressure comes as a result of the asymmetric contour of the airfoil.

3. For the **symmetric airfoil**, both the c.p. and a.c. actually **coincide at the quarter-chord point** (thin airfoil theory).

4. For the **cambered airfoil**, the location of the a.c. **depends on the lift and moment curve slopes**

5 Aerodynamic Center

Recall the sum of moments about aerodynamic center:

$$M_{ac} = M_{c/4} + Lx_{ac} \tag{15}$$

And define the dynamic pressure as:

$$q_\infty = \frac{1}{2}\rho_\infty u_\infty^2 \tag{16}$$

If we divide it by $q_\infty Sc$:

$$\frac{M_{ac}}{q_\infty Sc} = \frac{M_{c/4}}{q_\infty Sc} + \frac{L}{q_\infty S}\frac{x_{ac}}{c} \tag{17}$$

Here we define the **aerodynamic moment coefficient**:

$$\boxed{C_M = \frac{M}{q_\infty Sc}} \tag{18}$$

Then we have:

$$C_{M_{ac}} = C_{M_{c/4}} + C_L\frac{x_{ac}}{c} \tag{19}$$

Now we introduce a new term called **section aerodynamic coefficient.** Instead of describing the performance of a complete wing, this represents specifically to a section of **an airfoil**, independent of wing's span or the aircraft overall size. For example, **section lift coefficient:**

$$\boxed{c_l = \frac{l}{q_\infty c}} \tag{20}$$

Here:

1. l: lift force **per unit span**
2. c: chord length of the airfoil section

And similarly **section aerodynamic moment coefficient:**

$$\boxed{c_m = \frac{M}{q_\infty c}} \tag{21}$$

Notice here M is still the **total moment!** Therefore we can get the transformations:

$$\frac{c_m}{S} = \frac{M}{q_\infty S c} = C_M \tag{22}$$

$$\frac{c_l}{S} = \frac{L/b}{q_\infty S c} = C_L \tag{23}$$

Therefore we have the relation in terms of section coefficients:

$$c_{m_{ac}} = c_{m_{c/4}} + c_l \frac{x_{ac}}{c} \tag{24}$$

Differentiate with respect to **angle of attack** α:

$$\frac{dc_{m_{ac}}}{d\alpha} = \frac{dc_{m_{c/4}}}{d\alpha} + \frac{dc_l}{d\alpha}\frac{x_{ac}}{c} \tag{25}$$

Recall the definition of aerodynamic center, it is where aerodynamic moment is independent of the lift coefficient or angle of attack. Therefore:

$$\frac{dc_{m_{ac}}}{d\alpha} = 0 \tag{26}$$

Rearrange, we get:

$$\frac{dc_{m_{c/4}}}{d\alpha} = -\frac{dc_l}{d\alpha}\frac{x_{ac}}{c} \tag{27}$$

This relation could also be shown in graph:

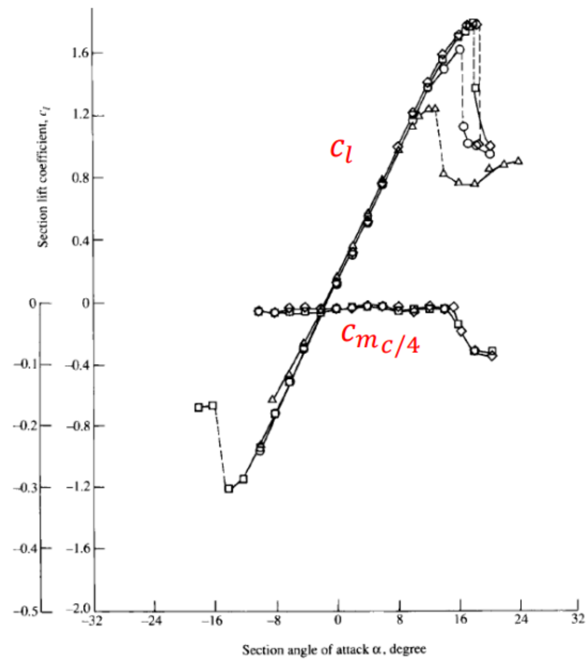

Figure 9: Relation between c_l and c_m

From observations, we can see that in linear portions two curves' slopes are constant **with respect to angle of attack**. Therefore we can solve the important parameter $\frac{x_{ac}}{c}$, which is **location of aerodynamic center with respect to airfoil chord**.

$$\frac{x_{ac}}{c} = -\frac{\frac{dc_{m_{c/4}}}{d\alpha}}{\frac{dc_l}{d\alpha}} = -\frac{m_0}{a_0} \tag{28}$$

Where:

$$a_0 = \frac{dc_l}{d\alpha} \tag{29}$$

a_0 refers to the **lift-curve** slope of an airfoil. For **thin airfoil in subsonic, incompressible flow**, $a_0 = 2\pi$.

m_0 refers to the **zero-lift pitching moment coefficient of an airfoil**. It is the value of c_m when the lift coefficient c_l is zero.