Aerodynamic Duct

1 Introduction

The main problem of air inlet design is to ensure that an aircraft engine is properly supplied with air under **all conditions of operation** and that the aptitude (quality) of the airframe is not unduly (severely) impaired in the process.

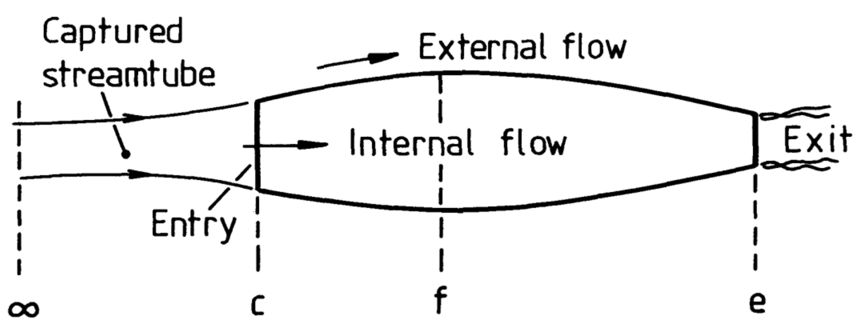

Figure 1: Aerodynamic Duct

To better study this problem, we introduce the concept of **aerodynamic duct**. The duct captures a certain **streamtube of air**, then divides the airstream into **internal flow** and an **external flow**.

1. Internal flow: feed the engine
2. External flow: preserve the good aerodynamics of the airframe

An engine requires to take in its air at a **moderate subsonic speed**, which is at a speed lower than the aircraft speed. Therefore the front part of the duct is in the form of a **diffuser**. The rear part of the duct is then **convergent**, representing the engine nozzle system.

2 Stations and Indices

1. Station ∞ : undisturbed flow, or free stream
2. Station c: duct entry
3. Station f: engine face position

4. Station e: duct exit
5. A_∞: stream tube area
6. A_c: Entry area, **first term of choice for the inlet designer**
7. A_f: internal cross-sectional area at station f, also the maximum area, which is fixed by the engine size.
8. A_e: the size of exit

3 Flow Quantities

Based on the continuity equation:

$$\rho_\infty U_\infty A_\infty = \rho_c U_c A_c = \rho_f U_f A_f = \rho_e U_e A_e \tag{1}$$

Assume **incompressible and constant density:**

$$U_\infty A_\infty = U_c A_c = U_f A_f = U_e A_e \tag{2}$$

In this chapter, we define the **total pressure** as P, then we can write:

$$P_\infty - p_e = (P_\infty - P_e) + (P_e - p_e) \tag{3}$$

On the right hand side:

1. The first term represents the change in **total pressure of the internal flow** ΔP, usually it is a positive value representing the pressure loss.
2. The second term is the **dynamic pressure (q_e) at the exit**, based on Bernoulli's Equation.
$$q_e = \frac{1}{2}\rho U_e^2 \tag{4}$$

Divide this equation by q_∞ and rearrange, we can get:

$$\frac{q_e}{q_\infty} = \frac{P_\infty - p_e}{q_\infty} - \frac{\Delta P}{q_\infty} = 1 - \frac{p_\infty - p_e}{q_\infty} - \frac{\Delta P}{q_\infty} \tag{5}$$

The second term on the right is defined as **static pressure coefficient at exit:**

$$C_{pe} = \frac{(p_\infty - p_e)}{q_\infty} \tag{6}$$

Based on **Joukowski condition** for the wing, we assume **the static pressures are equal in the internal and external flows on the two sides of the edge.** Generally pressure is not greatly different from free stream at infinity, so **value of C_{pe} is expected to be close to unity.** For the total pressure change (ΔP), it will depend on internal velocity (U^2) proportionally, so we assume:

$$\Delta P = kq_f = k\frac{1}{2}\rho U_f^2 = k\rho\frac{1}{2}\frac{U_e^2A_e^2}{A_f^2} = kq_e(\frac{A_e}{A_f})^2 \tag{7}$$

Plug back into the previous equation, we get:

$$\frac{q_e}{q_\infty}(1 + k\frac{A_e^2}{A_f^2}) = 1 - C_{pe} \tag{8}$$

Using this relation, we can also rewrite the ratio of stream tube area and exit area:

$$\frac{A_\infty}{A_e} = \frac{U_e}{U_\infty} = (\frac{q_e}{q_\infty})^{\frac{1}{2}} = (\frac{1 - C_{pe}}{1 + kA_e^2/A_f^2})^{\frac{1}{2}} \tag{9}$$

Some remarks:

1. Under incompressible conditions, the flow quantity through the empty duct is determined **primarily by the exit area.**
2. There is no dependence on **entry area**: the flow at entry adapts to the value determined by the exit.
3. **Larger entry** would take the same flow quantity at **lower velocity**. **Smaller entry** would take the same flow quantity at **higher velocity**.

4 Exit Area Dependence

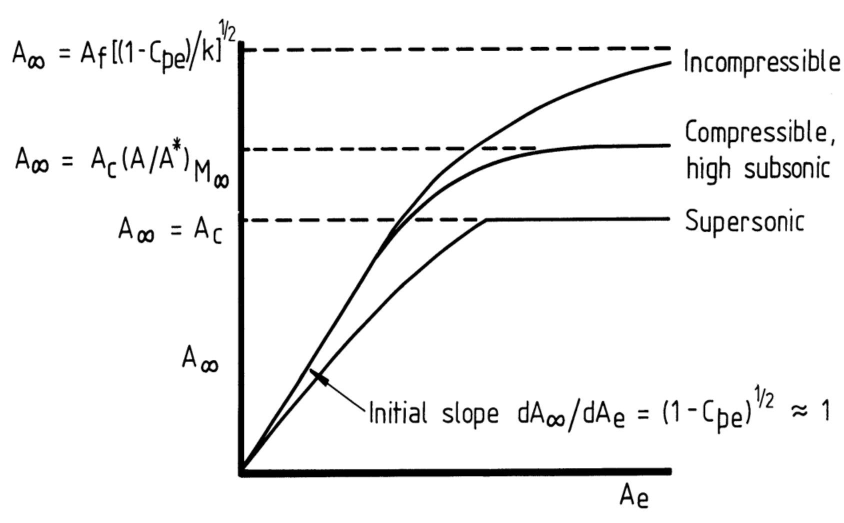

Figure 2: Exit Area Dependence

Now we want to investigate how exit area will affect other areas in inlet. Some observations from the graph:

1. **When A_e is small:** the term kA_e^2/A_c^2 is small, so the slope is approximately $((1-C_{pe})^{1/2})$, which is also close to unity. Therefore, **stream tube area approximately equal to the exit area.**

2. **As A_e increases but still not high enough to affect k and C_{pe}:** we need to start to consider k term. Then we have:

$$\frac{A_\infty}{A_f} = \frac{A_\infty}{A_e}\frac{A_e}{A_f} \approx (\frac{1-C_{pe}}{k})^{\frac{1}{2}} \tag{10}$$

3. **Once capture area A_∞ is greater than entry area A_c:** the flow has to accelerate from free stream into the entry. There will be a point **the Mach number at entry becomes unity**, so that the entry is choked and can not accept further increase. This actually occurs when:

$$\frac{A_\infty}{A_c} = \frac{A}{A^*} \tag{11}$$

which is the **sonic area ratio** at free stream Mach number.

4. **When the main stream is supersonic:** Both the **exit pressure** and **loss term** require reconsideration. The existence of **shock at the duct exit** will cause a difference between internal and external **static pressures**, which will affect the exit pressure. The loss term now must include **an allowance for shock loss in the inlet.** Also, **the capture area cannot exceed the entry area.**

5. **If we have an engine installed:** the temperature term must be included:

$$A_\infty = f(A_e, \Delta P, \Delta T) \tag{12}$$

For small flows, through the control exercised by the engine expressed by ΔP and ΔT may be more influential factor than the loss term alone.