Multidisciplinary Optimization (MDO)

1 Design

1.1 Definition

The intellectual engineering process of creating on paper a flying machine that meets certain requirements and performance objectives, or explores new concepts, technologies and innovations.

1.2 Characteristics

1. **Need:** The design process is both started by and constrained by an identified need.
2. **Non-Unique Solutions:** Many legitimate solutions will exist for the same requirements. The final accepted solution will always involve compromise and judgment.
3. **Systematic:** Because many solutions exist, it is necessary to use a systematic method to identify the "best" solution.
4. **Iterative:** The design process is iterative, and often requires returning to an earlier step if assumptions are found invalid.
5. **Interdisciplinary:** By its very nature, the design process involves considerations and compromises between a variety of disciplines. Designers must have more than a basic understanding of all of the disciplines involved, and understand how they interact.
6. **Teamwork:** Above all, the design of a complex system requires participation and disciplined communication by everyone involved.

2 Multi-Disciplinary Analysis Optimization (MDAO)

2.1 Why?

A paradigm shift in industry:

1. Integration of more disciplines early on to increase knowledge

2. Leveraging computing capabilities to maintain design freedom carrying more concepts forward
3. Decreasing the costs committed with limited information

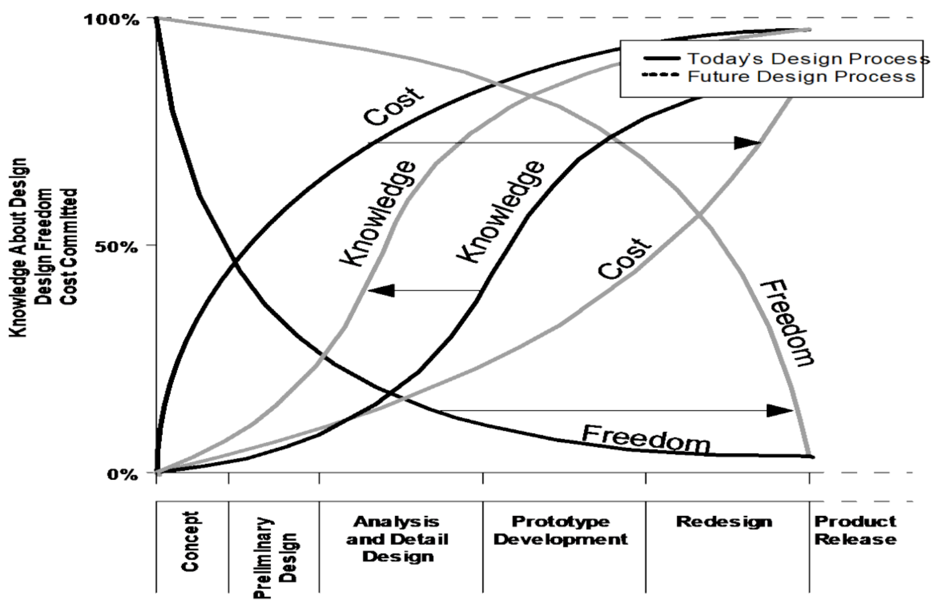

Figure 1: Paradigm Shift.

2.2 How?

MDAO is a branch of applied mathematics in the domain of optimization mainly.

1. Integrate high fidelity simulation tools
2. Process a huge number of variables and constraints
3. Take into account model uncertainties
4. Use efficient optimization techniques, advanced surrogate models and analysis integration frameworks.

3 Response Surface Methodology (RSM)

1. **RSM** is a technique for building and optimizing empirical models of continuous functions.
2. **RSM** is a multivariate linear regression technique developed to model the response of a complex system using a simplified equation.
3. Regression data is obtained intelligently through Design of Experiments (DoE) techniques.

4 Design of Experiment (DOE)

4.1 Key Concepts

1. **Replication:** Repeating trials or measurements; useful if the results have inherent noise. Typically not used in simulation.
2. **Correlation:** Non-independence of input variables
3. **Orthogonality:** Implies zero correlation between experimental factors.
4. **Blocking:** Arranging experimental units into groups that are similar to one another to reduce known but irrelevant sources of variability and focus on the estimation of study parameters.
5. **Factorial Design:** Allows the effect of several factors and their interactions to be determined with the same number of trials as are needed to determine any single effect.

4.2 Methods (Brief)

1. Full Factorial Design
2. Fractional Factorial Design
3. Face-centered Central Composite Design
4. Box-Behnken Design
5. Space Filling Design

4.3 How to Choose DOE for RSM?

Considerations include:

1. Points at the extremes
2. The number of variables or factors
3. Speed (or execution time) of the analysis tools
4. The overall accuracy desired
5. The behavior of the response
6. Convergence behavior of the modeling tool(s)

4.4 Summary

1. Design-of-Experiments is a proven process for maximizing the information gained from computer experiments or physical designs.
2. As opposed to full parameter sweeps (full-factorial), DoE changes several factors simultaneously in a statistically valid manner.
3. DoE can be used for characterization, optimization, or design space exploration to increase problem understanding.
4. DoE can also be used to build predictive models using response surface methodology or other surrogate techniques.

5 The Approximating Function

5.1 Advantages

1. **Complexity:** It may not be possible to represent the physical mechanism as a function. Approximating function is much simpler than the true function.
2. **Knowledge:** The physics may not be fully understood. The mechanism can be stochastic itself so finding the true function is impossible. The experimenter may not have the resources for a full-scale scientific study.
3. **Scope:** Experimenter may be interested in the effects of a few parameters. Experimenter may be interested in the process in a small region.
4. **Practicality:** Finding regression coefficients is relatively easy. Flexible approximating functions exist which can approximate most natural phenomena.

5.2 First Order Approximating Function

$$\eta = \beta_0 + \beta_1 x_1 + ... + \beta_k x_k \tag{1}$$

The parameters β_j are called regression coefficients. This is called the main effects model because it includes only the main effects of the coded variables.

5.3 Interaction Terms

$$\eta = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \beta_k x_k \tag{2}$$

Interaction terms introduce curvature on the approximating function. Strength of interaction determines the amount of curvature. If the interaction is reinforcing, concave up; else if interaction is conflicting, concave down.

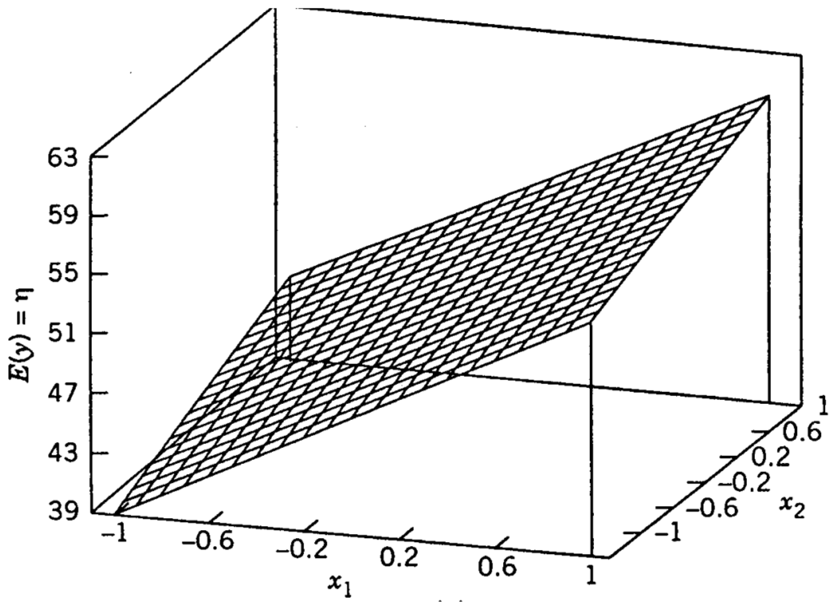

Figure 2: First Order Approximation.

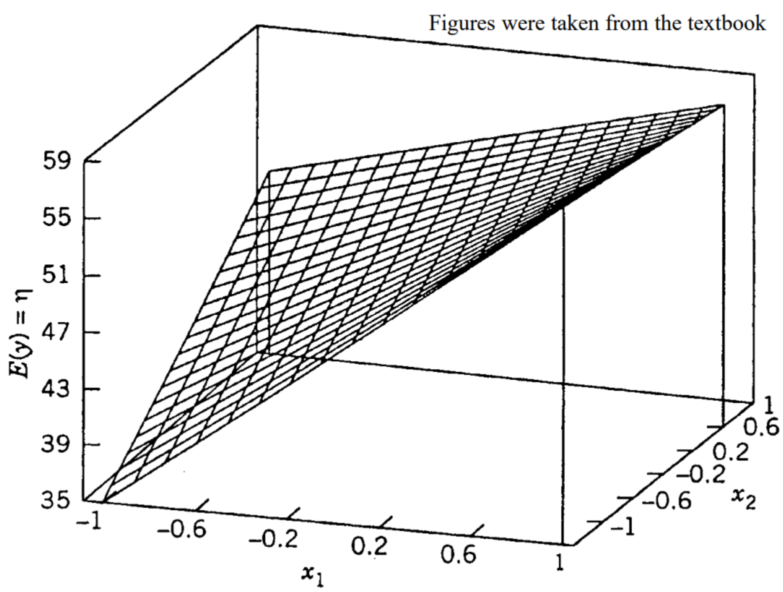

Figure 3: Approximation with Interactions.

6 Model Adequacy Checking

6.1 R^2

1. A mathematical measure that estimates how well the assumed functional form of the response measures the variability of the supplied response data.

2. A perfect fit of the response data corresponds to an R^2 value of 1.0

3. A high value of R2 does not indicate anything about the goodness of the resulting

model, but a low value of R2 is highly indicative of a problem Response model.

6.2 Actual by Predicted Plot

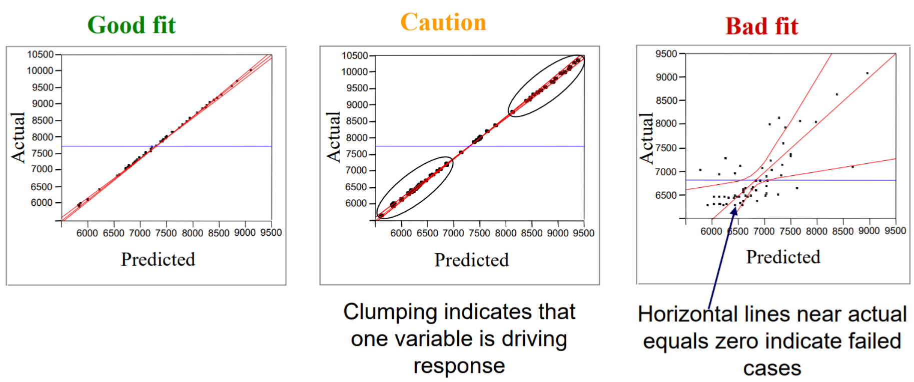

Figure 4: Actual by Predicted Plot.

6.3 Residual

1. The error in the fitted model, and is the difference between the actual value of each observation and the value predicted by the fitted model.

2. Residuals are elements of variation unexplained by the regression model generated.

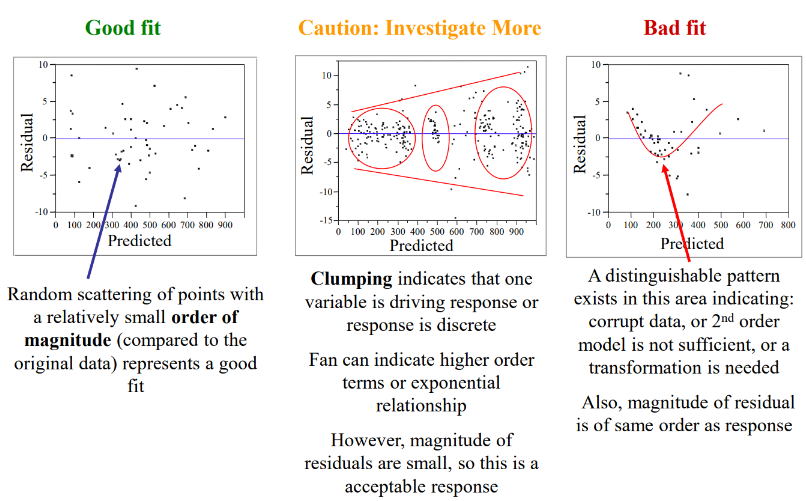

Figure 5: Residual Plot.

6.4 Model Fit Error (MFE)

MFE is the relative error of the model with respect to the actual values, measured on the points used to create the model.

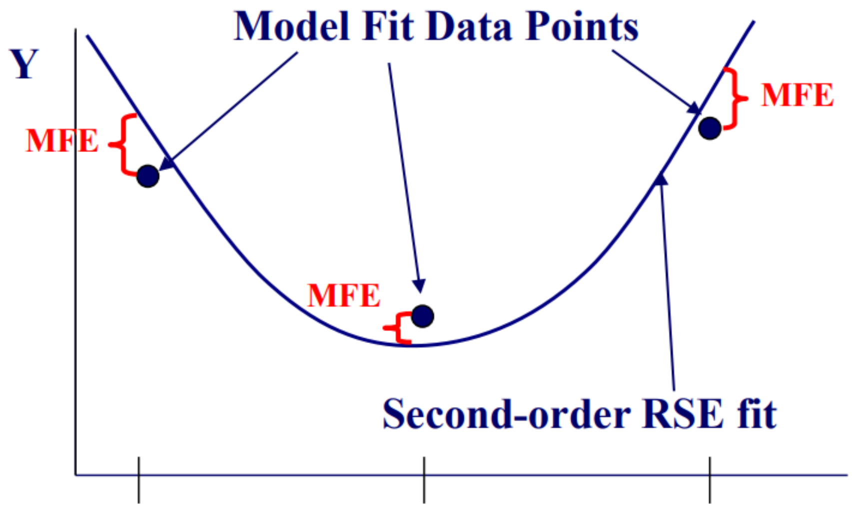

Figure 6: MFE Plot.

6.5 Model Representation Error (MRE)

Least squares regression attempts to fit the supplied DoE data points as best as possible.

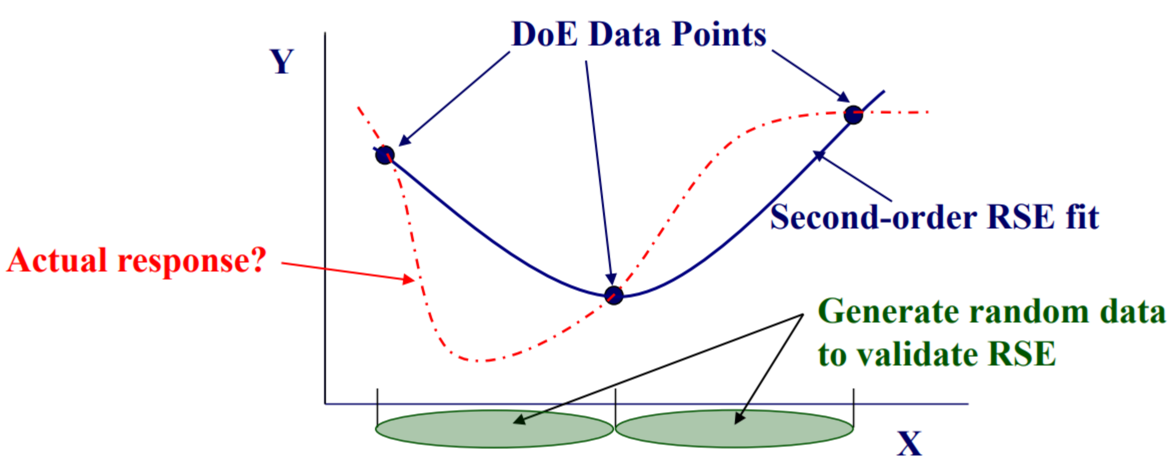

Figure 7: MRE Plot.