Mach Wave and Mach Angle

1 Mach Wave and Angle

We already know the definition of the speed of sound, now we are more interested in **relationship bewtween speed of sound and flow speed**, or speed of body moving through fluid.

Consider a small body moving in **stagnant fluid, continuously produces weak pressure disturbances**. The disturbances travel outward spherically at sound speed. Now we want to look at disturbances generated **at equally spaced time intervals**.

1.1 If $u << a$

Then the body is nearly stationary, the propagation is nearly a perfect circle:

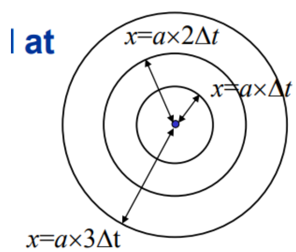


Figure 1: Very Low Speed Flow

1.2 If $u < a$

This is the subsonic case, subsonic body always behind sound waves launched from previous positions.

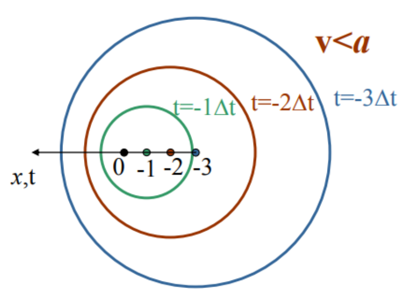


Figure 2: Subsonic Flow

1.3 If $u > a$

This is the supersonic case. Supersonic body moves ahead of previous sound waves.

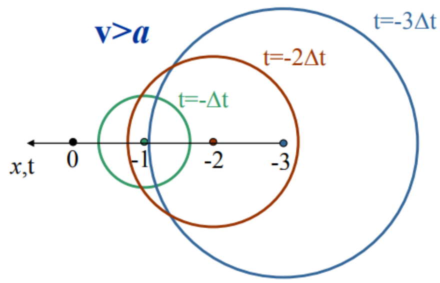


Figure 3: Supersonic Flow

For supersonic flow, we can define region **where disturbance has had an effect**. This is a conical region **delineated by tangents to sound wave spheres.**

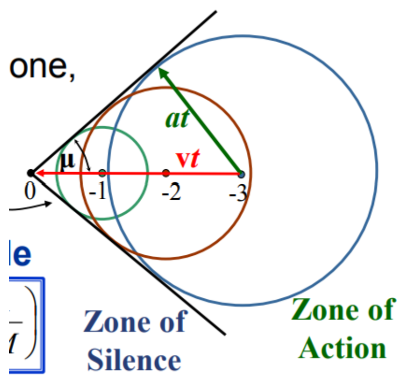


Figure 4: Mach Wave

Waves **coalesce** at the edge of cone, produce largest disturbance. Therefore we call the edge as **Mach wave or Mach line**. And the **Mach angle** is defined as the angle between Mach line and body motion:

$$\mu = sin^{-1}(\frac{at}{ut}) = sin^{-1}(\frac{1}{M}) \tag{1}$$

2 Shock Waves

If we let the body be stationary, and the flow is moving, we can get the same behavior:

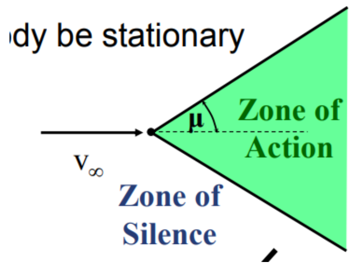


Figure 5: Weak Wave Case

The weak disturbances from presence of body can **only be felt inside Mach cone, but not upstream**.

If the body has finite size, then **strong (nonisentropic)** pressure disturbances can occur, **they coalesce to form shock waves**. The shock angle $\beta > \mu$.

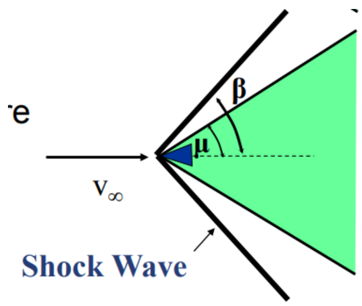


Figure 6: Strong Wave Case

3 Adiabatic Flow Ellipse

Based on energy conservation:

$$h_o = h + \frac{u^2}{2} = const \tag{2}$$

For the TPG and CPG, stagnation temperature also constant:

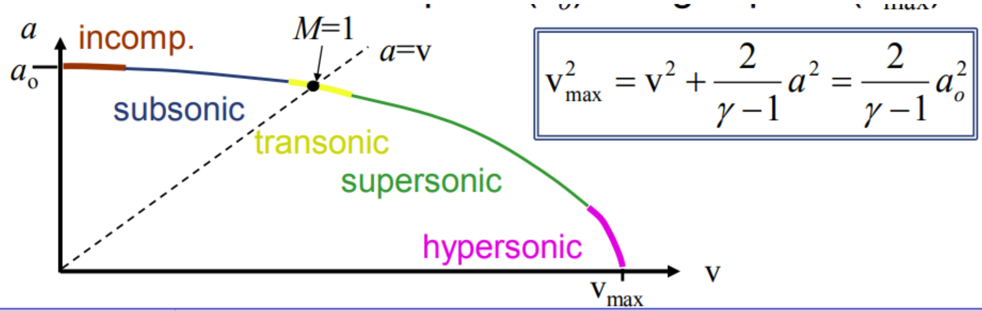
$$T_o = T + \frac{u^2}{2c_p} = T + \frac{\gamma - 1}{\gamma R}\frac{u^2}{2} \tag{3}$$

$$\frac{2}{\gamma - 1}\gamma RT + u^2 = const \tag{4}$$

$$\frac{2}{\gamma - 1}a^2 + u^2 = u_{max}^2 = \frac{2}{\gamma - 1}a_o^2 \tag{5}$$

Where a_o is the **stagnation speed of sound, with no kinetic energy left** **(**$u = 0$**)**. And u_{max} is the **maximum velocity possible, no thermal energy left** **(**$T = 0$**)**.

Drawing the transition from low speed (a_o) to high speed u_{max}, we can get the **adiabatic flow ellipse**:



Regime	Description/Interpretation
incomp.	$v \ll a$, $da \ll dv$, little change in $a(T)$
subsonic	$v \lesssim a$, M changes primarily to changes in v
transonic	$\lvert v - a \rvert \ll v, a$
supersonic	$v > a$, M changes through substantial changes in v and $a(T)$
hypersonic	$v \gg a$, $dv \ll da$, M change mostly due to $a(T)$ changes

Figure 7: Adiabatic Flow Ellipse