

Oblique Shock

1 Overview

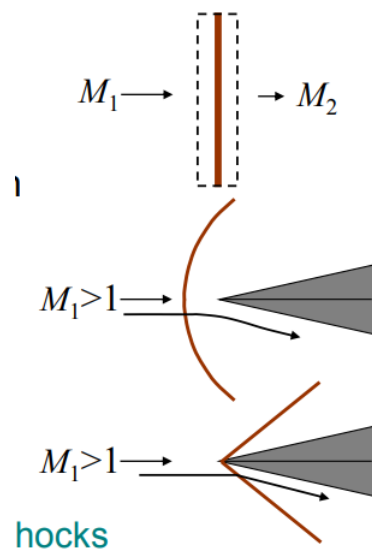


Figure 1: Shock Turning

From the previous chapter, we know for normal shocks, flow is perpendicular to shock, so there is no change in flow direction. However, 'normally' it is not the case, the flow could change its direction:

1. **Bow shock:** The flow is **slow to subsonic ahead of turn**, so there will be a gradual turn.
2. **Oblique shock:** Go through non-normal wave with **sudden angle change**.

2 Definition

Let's start with Mach wave towards **infinitely thin body**, then now no flow turn is required. The result is **infinitesimal weak wave**:

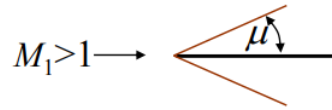


Figure 2: Mach Wave

Recall the definition of **Mach angle**:

$$\mu = \sin^{-1}\left(\frac{1}{M}\right) \quad (1)$$

Now we consider a finite-sized wedge, with **half angle (turning angle)** δ . The flow must undergo compression to turn, which will create an oblique shock with **shock angle** θ :

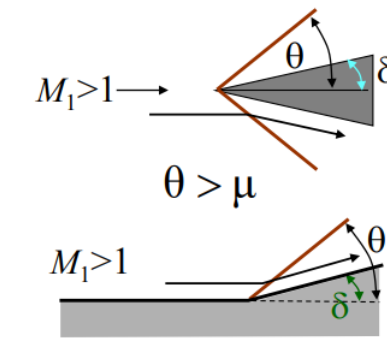


Figure 3: Shock Angle

3 Governing Equations

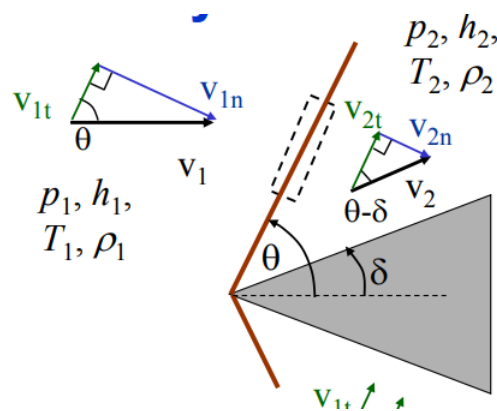


Figure 4: Decomposition

Now we decompose the velocity to two components:

1. u_t : one tangent to **shock (not the surface!)**

2. u_n : one normal to **shock** (**not the surface!**)

Then along the shock, we have:

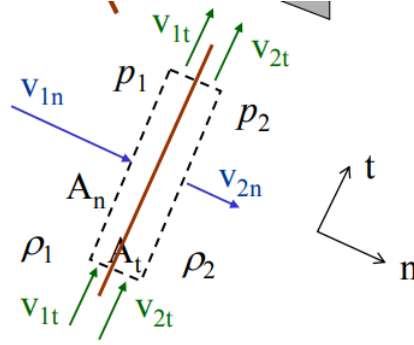


Figure 5: Shock View

$$u_{1n} = u_1 \sin \theta, \quad u_{1t} = u_1 \cos \theta \quad (2)$$

$$u_{2n} = u_2 \sin(\theta - \delta), \quad u_{2t} = u_2 \cos(\theta - \delta) \quad (3)$$

Similarly:

$$M_{1n} = M_1 \sin \theta, \quad M_{1t} = M_1 \cos \theta \quad (4)$$

$$M_{2n} = M_2 \sin(\theta - \delta), \quad M_{2t} = M_2 \cos(\theta - \delta) \quad (5)$$

One important characteristic of oblique shock is that **the tangent component of velocity will not change!** Which means:

$$\boxed{u_{1t} = u_{2t}} \quad (6)$$

Using this relation, we can write the **oblique shock relations** in terms of **normal shock relations with normal component of velocity**.

Mass Conservation:

$$\boxed{\rho_1 u_{1n} = \rho_2 u_{2n}} \quad (7)$$

Momentum Conservation:

$$\boxed{p_1 + \rho_1 u_{1n}^2 = p_2 + \rho_2 u_{2n}^2} \quad (8)$$

Energy Conservation:

$$\boxed{h_1 + \frac{u_{1n}^2}{2} = h_2 + \frac{u_{2n}^2}{2}} \quad (9)$$

However, even though u_t is constant, M_t **is not**:

$$\boxed{\frac{M_{2t}}{M_{1t}} = \frac{u_{2t}/a_2}{u_{1t}/a_1} = \sqrt{\frac{T_1}{T_2}}} \quad (10)$$

4 Static Properties Relations

Now we explore the relations between the Mach number. Recall the normal shock relations, we have:

$$M_{2n}^2 = \frac{M_{1n}^2 + \frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1}M_{1n}^2 - 1} \quad (11)$$

Replace the normal component by the actual velocity:

$$M_2^2 \sin^2(\theta - \delta) = \frac{M_1^2 \sin^2 \theta + \frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1}M_1^2 \sin^2 \theta - 1} \quad (12)$$

To simplify the equation, we still use M_{1n} . Therefore the static property relations include (all related to [normal shock relations](#)):

$$\frac{p_2}{p_1} = \frac{2\gamma}{\gamma+1}M_{1n}^2 - \frac{\gamma-1}{\gamma+1} \quad (13)$$

$$\frac{u_{2n}}{u_{1n}} = \frac{(\gamma-1)M_{1n}^2 + 2}{(\gamma+1)M_{1n}^2} \quad (14)$$

$$\frac{T_2}{T_1} = \frac{(1 + \frac{\gamma-1}{2}M_{1n}^2)}{(1 + \frac{\gamma-1}{2}M_{2n}^2)} \quad (15)$$

5 Stagnation Properties Relations

Similarly, we can get the relations of stagnation properties. Due to energy conservation, we still have:

$$T_{o2} = T_{o1} \quad (16)$$

And from normal shock relations, we have:

$$\frac{p_{o2}}{p_{o1}} = \left[\frac{\frac{\gamma+1}{2}M_{1n}^2}{1 + \frac{\gamma-1}{2}M_{1n}^2} \right]^{\frac{\gamma}{\gamma-1}} \left[\frac{2\gamma}{\gamma+1}M_{1n}^2 - \frac{\gamma-1}{\gamma+1} \right]^{\frac{1}{1-\gamma}} \quad (17)$$

6 Wave/Shock Angle

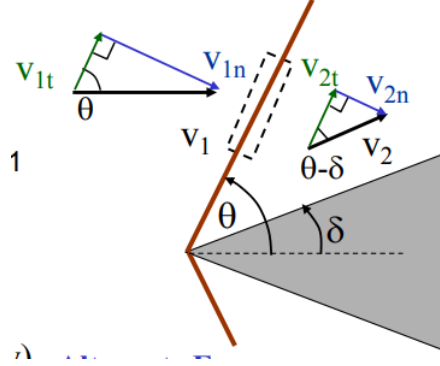


Figure 6: Angle Calculation

In previous section, we always use M_1 and **shock angle** (θ) to calculate the properties relations. However, usually we only know M_1 and **turning angle** δ . So we want to know the relations between these two angles. From geometry:

$$\frac{u_{2n}}{u_{1n}} = \frac{u_{2t} \tan \theta}{u_{1t} \tan (\delta - \theta)} = \frac{(\gamma - 1)M_{1n}^2 + 2}{(\gamma + 1)M_{1n}^2} \quad (18)$$

Again, after some magic math, we have:

$$\tan \delta = \frac{(2/\tan \theta)(M_1^2 \sin^2 \theta - 1)}{M_1^2(\gamma + \cos 2\theta) + 2} \quad (19)$$

Alternatively:

$$\tan \delta = \frac{(1/\tan \theta)(M_1^2 \sin^2 \theta - 1)}{\frac{\gamma+1}{2}M_1^2 - (M_1^2 \sin^2 \theta - 1)} \quad (20)$$

Therefore, normally there are three ways to get the solution:

1. Using M_1 and θ
2. Using M_1 and δ
3. Using δ and θ

Notice that these equations are quadratic equations, so for given (M_1, δ) , **there are so solutions**. Normally we check the **oblique shock chart**:

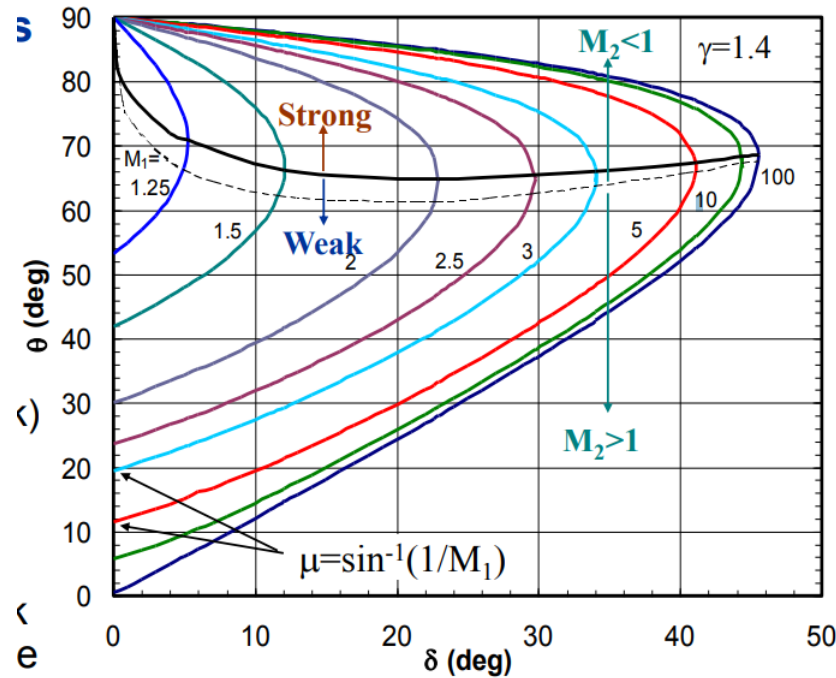


Figure 7: Oblique Shock Chart

Some characteristics of **weak shocks**:

1. Usually $M_2 > 1$
2. $\theta_{min} = \mu$
3. θ is smaller than strong shock

Some characteristics of **strong shocks**:

1. $\theta_{max} = 90^\circ$, which corresponds to normal shock
2. Always $M_2 < 1$

7 Procedures to Solve Oblique Shock

1. Given M_1 and δ , using the equation or checking the oblique shock chart to get θ
2. Calculate $M_{1n} = M_1 \sin \theta$
3. Use normal shock tables or oblique shock relations to find the property ratios
4. Get M_2 from $M_2 = M_{2n} / \sin(\theta - \delta)$

8 Strong and Weak Oblique Shocks

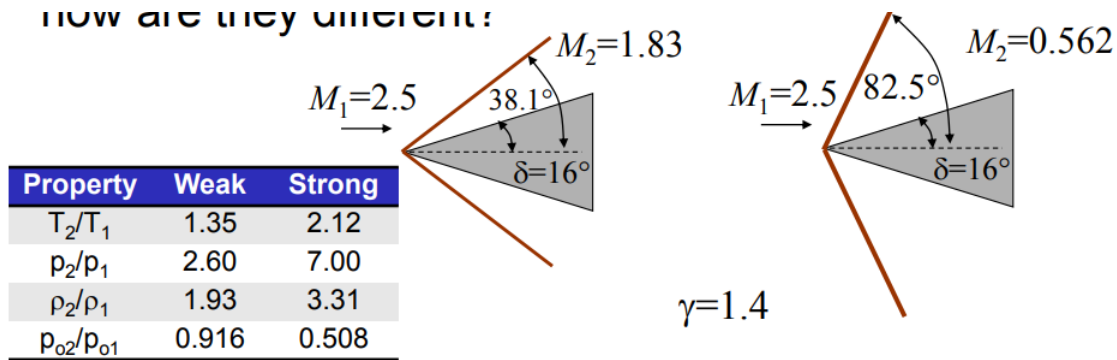


Figure 8: Weak and Strong Oblique Shocks

Why with same turn angle, there are two solutions?? From the graph we can see that **stronger shock (closer to normal shock) produces more change**. But we want to know what determines which solution will occur.

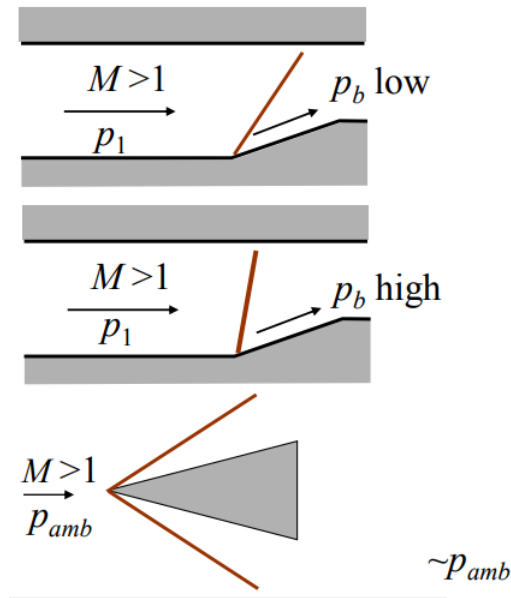


Figure 9: Pressure Dependence

Based on the experiments, this actually depends on **upstream versus downstream pressure**. If the back pressure is high, the shock will be stronger.

For the **internal flow**, we can have p_2 close to or much higher than p_1 , so usually there will be a strong shock. For the **external flow**, both pressures close to ambient pressure, so usually we get weak shock.

9 Reflected Oblique Shock

9.1 Definition

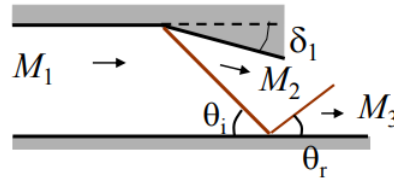


Figure 10: Reflected Oblique Shock

Consider 'Weak' ($M_2 > 1$) oblique shock wave impinging on a flat wall. Three steps will happen:

1. Incident shock wave turns flow toward the lower wall.
2. Flow can not pass through boundary, **must turn back parallel to lower wall** (δ_1)
3. Flow turns back on itself, which will cause **compression**. Therefore, the reflected wave is **oblique shock**

Notice that the shock is weaker than incident shock because $M_2 < M_1$.

9.2 Mach Reflection

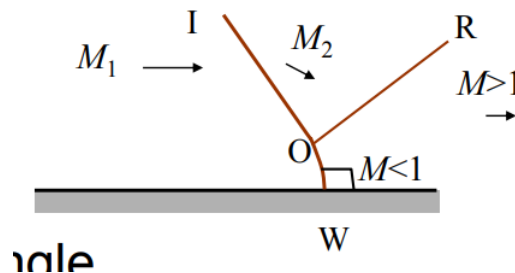


Figure 11: Mach Reflection

If M_2 low enough, then required turning angle for reflected wave may **exceed maximum oblique shock angle**, this case **no simple reflected wave possible**. Instead, we will get something like detached shock

In the graph:

1. IO: incident oblique shock
2. OW: strong curved shock
3. OR: weak oblique shock

9.3 Pressure BC

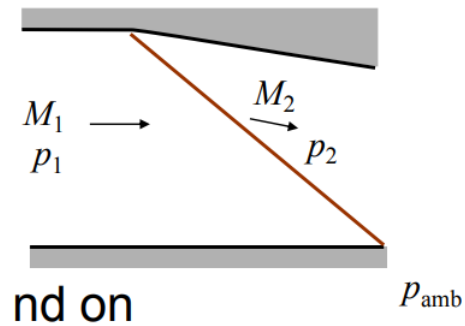


Figure 12: Pressure BC

If oblique shock hits a pressure boundary condition (like ambient pressure), reflected wave must **adjust flow pressure to match boundary pressure**. Type of reflected wave will depend on whether pressure must drop or rise:

1. If pressure needs to rise: compression, flow will turn back on itself
2. If pressure needs to drop: expansion, flow will open up