Reference Frame

1 Points of View

1.1 Continuum View

The continuum view is based on the assumption that the fluid is a continuous medium, with no gaps between its molecules. It neglects the discrete nature of molecules and considers the fluid as a continuous distribution of mass, momentum, and energy. This view allows us to use mathematical techniques such as calculus and partial differential equations to describe fluid properties like pressure, temperature, and velocity as continuous functions of space and time. The continuum view is suitable for many practical engineering problems where the fluid's discrete molecular structure has little to no effect on the problem's overall behavior. Most classical fluid mechanics equations, like the Navier-Stokes equations and the Euler equations, are derived based on the continuum view.

1.2 Microscopic View

The microscopic view considers the discrete nature of fluid molecules, taking into account their individual motion, interactions, and energy states. This approach is necessary when the continuum assumption breaks down, such as in cases of very low-density fluids, very small scales, or very high gradients. In these situations, fluid properties like pressure, temperature, and velocity can no longer be described as continuous functions of space and time, and the effects of individual molecules must be considered. The microscopic view involves statistical mechanics, molecular dynamics simulations, and the Boltzmann equation to describe fluid behavior on a molecular level.

2 Reference Frame

2.1 Eulerian

In the Eulerian reference frame, the observer is fixed at a specific point in space, and fluid properties such as velocity, pressure, and temperature are described as functions of both time and spatial coordinates (x, y, z). In this approach, the focus is on analyzing the changes in fluid properties at fixed locations within the fluid domain as the fluid flows through those locations. The Eulerian reference frame is widely used in fluid mechanics, particularly when dealing with complex geometries and flow fields. The Navier-Stokes equations, which describe fluid motion, are typically written in the Eulerian reference frame.

2.2 Lagrangian

In the Lagrangian reference frame, the observer moves along with a specific fluid particle or a set of fluid particles. Fluid properties are described as functions of time only, as the observer's position changes with the fluid particle's motion. The Lagrangian approach focuses on tracking individual fluid particles and analyzing the changes in their properties as they move through the flow field. The Lagrangian reference frame can be useful for understanding the motion of individual particles or the dispersion and mixing of substances within the flow, such as pollutants or dye.

2.3 Comparison

Notice that Lagrangian reference frame could be used in both continuum view and microscopic view. Within continuum view, the observer moves along with a specific fluid "parcel" or a set of fluid parcels, and fluid properties are described as functions of time only, as the observer's position changes with the fluid parcel's motion. The Lagrangian approach allows for tracking the motion of fluid parcels and analyzing the changes in their properties as they move through the flow field, while still assuming the fluid to be a continuum.

3 Fluid Visualization

3.1 Streamlines

Line or curve everywhere tangential to the velocity vector (Fig. 1, [1]). The stream function along a given streamline is a constant. Notice that streamlines are based on instantaneous Eulerian velocity fields. If the flow is unsteady, the streamlines will change with time.

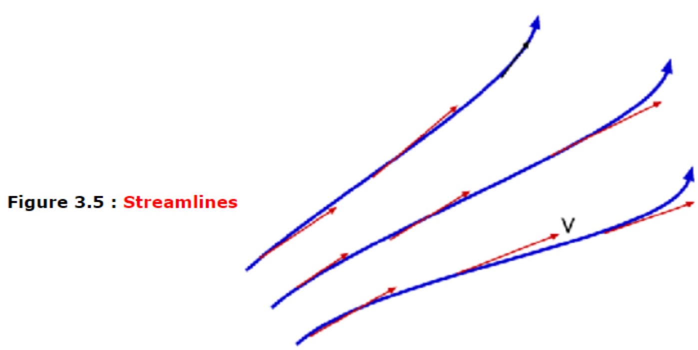

Figure 1: Streamlines

3.2 Pathlines

Path traced out in space over a finite period of time by the trajectory of a fluid particle (Fig. 2, [1])

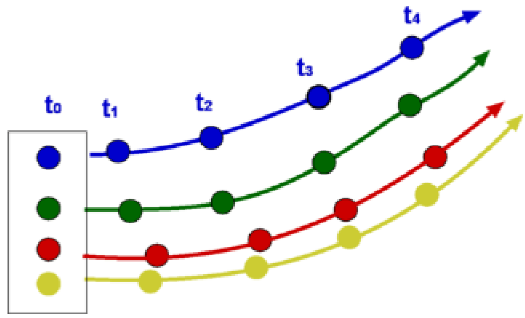

Figure 2: Pathlines

3.3 Streaklines

The locations at a chosen time of all fluid particles that passed through a fixed point at some previous time instant (Fig. 3, [1])

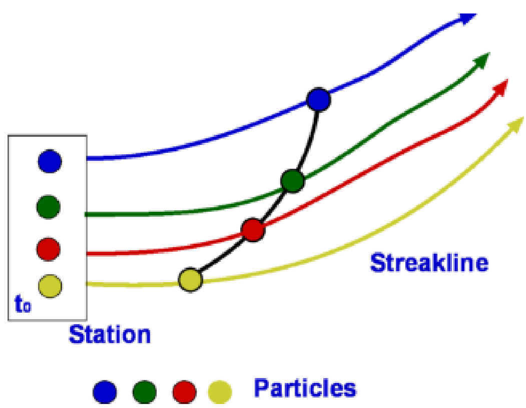

Figure 3: Streaklines

4 Material Derivative

4.1 Definition

The material derivative (also known as the substantial, convective, or total derivative) is a concept that connects the Lagrangian and Eulerian reference frames in fluid mechanics. It is formulated within the Eulerian reference frame, but it accounts for the motion of fluid parcels, which is a Lagrangian concept. It describes the rate of change of a fluid property for a specific fluid parcel as it moves through the flow field.

The material derivative considers both the local rate of change of a fluid property at a fixed location (the partial derivative with respect to time) and the change in the fluid property as the fluid parcel moves through the flow field (the convective term).

4.2 Derivation

Assume $\underline{\boldsymbol{y}} = \underline{\boldsymbol{x}}^+(0), t = 0$ be the initial position of the fluid particle, and f is any smooth, differentiable flow property. Then, the value of f at the instantaneous particle position at any time t is:

$$f^+(\underline{\boldsymbol{y}}, t) = f(\underline{\boldsymbol{x}}^+(\underline{\boldsymbol{y}}, t), t) \tag{1}$$

Then, apply the chain rule and "total differential" to get the rate of change following the fluid particle (ffp):

$$\begin{aligned} \frac{df^+}{dt} &= \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x_1}\frac{dx_1^+}{dt} + \frac{\partial f}{\partial x_2}\frac{dx_2^+}{dt} + \frac{\partial f}{\partial x_3}\frac{dx_3^+}{dt} \\ &= \frac{\partial f}{\partial t} + u_1\frac{\partial f}{\partial x_1} + u_2\frac{\partial f}{\partial x_2} + u_3\frac{\partial f}{\partial x_3} \end{aligned} \tag{2}$$

We use D/Dt as the material derivative operator, then we get:

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + (\underline{\boldsymbol{u}} \cdot \nabla) f \tag{3}$$

References

[1] URL http://www-mdp.eng.cam.ac.uk/web/library/enginfo/aerothermal_dvd_only/aero/fprops/cvanalysis/node8.html.