Transition

1 Overview

Transition to turbulence in fluid dynamics refers to the process through which a fluid flow changes from a laminar flow to a turbulent flow.

Laminar flow is a flow regime characterized by high momentum diffusion and low momentum convection. It occurs when a fluid flows in parallel layers, with no disruption between them, typically at low velocities. On the other hand, turbulent flow is a flow regime characterized by chaotic, stochastic property changes including low momentum diffusion, high momentum convection, and rapid variation of pressure and velocity in space and time.

2 Transition Process

The transition process in quiet boundary-layer flow past a smooth surface consists of the following processes:

1. Stable laminar flow near the leading edge
2. Unstable 2D Tollmien-Schlichting waves
3. Development of 3D unstable waves and hairpin eddies
4. Vortex breakdown at regions of high localized shear
5. Cascading vortex breakdown into fully 3D fluctuations
6. Formation of turbulent spots at locally intense fluctuations
7. Coalescence of spots into fully turbulent flow

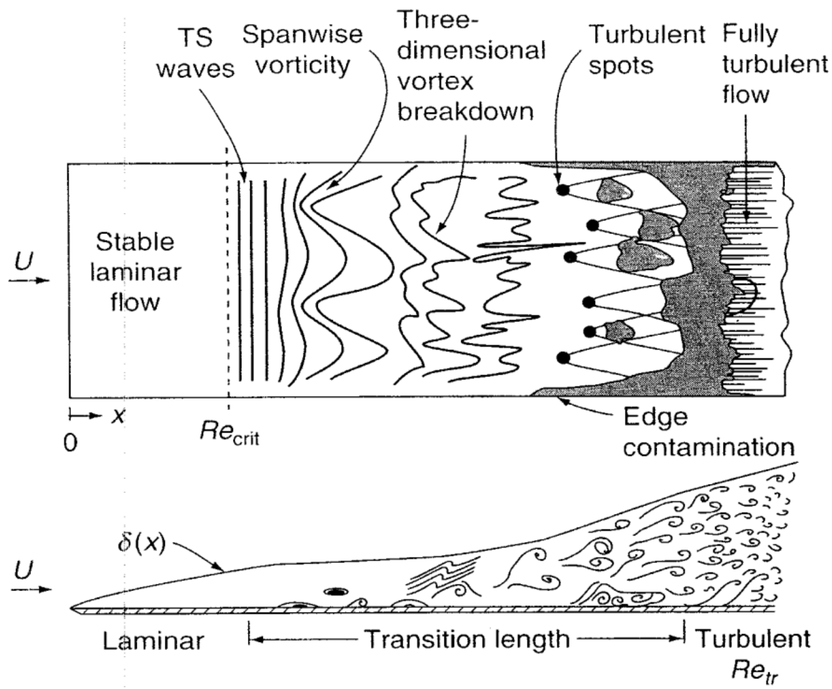

Figure 1: Transition.

3 Transition Control

3.1 Suction

Recall the boundary layer equation:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dP}{dx} + \nu\frac{\partial^2 u}{\partial y^2} \tag{1}$$

At the wall, due to no-slip condition, $y = 0, u = 0$, therefore:

$$v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dP}{dx} + \nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$\nu\frac{\partial^2 u}{\partial y^2} = \frac{1}{\rho}\frac{dP}{dx} + v\frac{\partial u}{\partial y} \tag{3}$$

Recall the stability chapter:

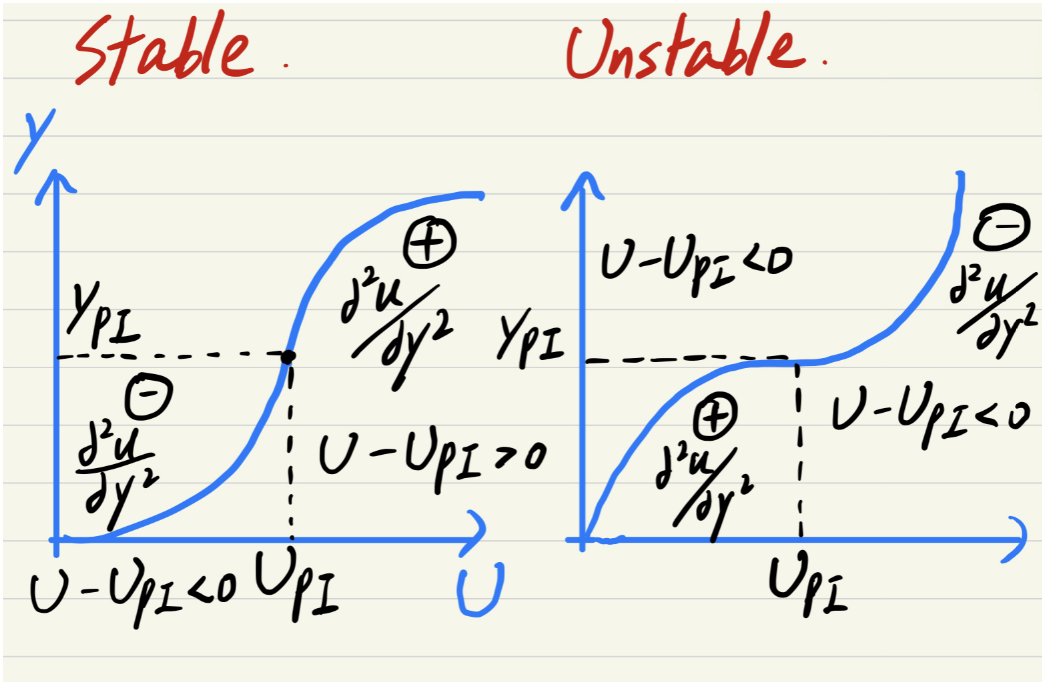

Figure 2: Stability.

At the near wall region, the more negative $\partial^2 u/\partial y^2$ is, the more stable the flow is. Therefore, we want $v < 0$ at the wall, which is called suction.

3.2 Change the Wall Temperature

Assume at the wall:

$$v = v_w, \mu = \mu_w \tag{4}$$

Recall the boundary layer equation at the wall, but now the viscosity is not a constant:

$$\rho v_w \frac{\partial u}{\partial y} = -\frac{dP}{dx} + \frac{\partial}{\partial y}(\mu \frac{\partial u}{\partial y}) \tag{5}$$

$$\rho v_w \frac{\partial u}{\partial y} = -\frac{dP}{dx} + \mu_w \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y}\frac{\partial u}{\partial y} \tag{6}$$

Rearrange the equation:

$$\mu_w \frac{\partial^2 u}{\partial y^2} = \frac{dP}{dx} + [\rho v_w - \frac{\partial \mu}{\partial y}]\frac{\partial u}{\partial y} \tag{7}$$

$$\mu_w \frac{\partial^2 u}{\partial y^2} = \frac{dP}{dx} + [\rho v_w - \frac{\partial \mu}{\partial T}\frac{\partial T}{\partial y}]\frac{\partial u}{\partial y} \tag{8}$$

Again, if we want to make the flow stable, we want negative $\partial^2 u/\partial y^2$, therefore we want:

$$\frac{\partial \mu}{\partial T}\frac{\partial T}{\partial y} > 0 \tag{9}$$

Recall the viscosity chapter, gas viscosity increases with the increase of temperature:

$$\frac{\partial \mu}{\partial T} > 0 \tag{10}$$

Therefore we also want:

$$\frac{\partial T}{\partial y} > 0 \tag{11}$$

Which means the far wall region should have higher temperature than near wall region, so we need to cool the wall to make the gas flow stable.

Similarly, liquid viscosity decreases with the increase of the temperature:

$$\frac{\partial \mu}{\partial T} < 0 \tag{12}$$

Therefore we also want:

$$\frac{\partial T}{\partial y} < 0 \tag{13}$$

Which means the far wall region should have lower temperature than near wall region, so we need to heat the wall to make the liquid flow stable.