## Viscosity

### 1 Definition

Viscosity is a property of the fluid, representing momentum exchange between molecules. Molecules always have random movements and collisions with each other. During the collisions, the momentum is exchanged.



Figure 1: Viscosity Explanation

As shown in the graph Fig. 1, the upper layer is faster than the lower layer, if this fluid has higher viscosity, it means the momentum exchange between these two layers is large, the faster velocity is decreasing significantly, which can explain the sentence: The viscosity of a fluid is a measure of its resistance to deformation at a given rate in Wikipedia. (Deformation of the fluid is caused by the difference of the velocity, in the blue block, the upper part is moving faster than the bottom, which will create a deformation)

### 2 Newtonian Fluid

#### 2.1 Normal Stress and Shear Stress

**Normal Stress:** When a force acts perpendicular to the surface of an object, it exerts a normal stress.

**Shear Stress:** When a force acts parallel to the surface of an object, it exerts a shear stress.

The unit of stress is the same as that of pressure, which is

$$\frac{force}{area} = \frac{ma}{A} = \frac{[kg][m/s^2]}{[m^2]} = [kg/(m \cdot s^2)] = [Pa]$$
(1)



Figure 2: Normal Stress and Shear Stress[1].

#### 2.2 Definition of Strain Rate

Strain rate is the change of strain (deformation) with respect to the time. As the ratio of two length, strain is dimensionless, which means the unit of strain rate  $\left(\frac{\partial u}{\partial x}\right)$  is  $s^{-1}$ .

**Normal Strain Rate**  $\frac{\partial u_i}{\partial x_i}$ : The strain rate of normal strain, which will change the volume of the fluid, including expanding or shrinking.

**Shear Strain Rate**  $\frac{\partial u_i}{\partial x_j}$ : The strain rate of shear strain , which is the deformation without changing the volume of the fluid.

### 2.3 Definition of Newtonian Fluid

A Newtonian fluid is a fluid whose shear stress is proportional to the shear strain rate during deformation, which can be expressed in tensor form:

$$\tau_{ij} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{2}$$

where  $\tau_{ij}$  is the shear stress tensor,  $\mu$  is the coefficient of viscosity of a Newtonian fluid.

### **3** Coefficients of Viscosity

### 3.1 Dynamic and Kinematic Viscosity

**Dynamic Viscosity:** Also known as absolute viscosity, it is the math relation between applied stress and local deformation of fluid.

$$\mu = \tau / \frac{\partial u}{\partial y} = \frac{[kg/(m \cdot s^2)]}{[s^{-1}]} = [kg \cdot m^{-1} \cdot s^{-1}]$$
(3)

**Kinematic Viscosity:** Dynamic viscosity divided by density. *Kinematic* usually refers to the study of motion, dividing viscosity by density is getting rid of the effect from mass.

$$\nu = \frac{\mu}{\rho} = \frac{[kg \cdot m^{-1} \cdot s^{-1}]}{[kg \cdot m^{-3}]} = [m^2 \cdot s^{-1}]$$
(4)

### 3.2 How does the viscosity change?

#### 3.2.1 Temperature

For liquid: Within the liquid, the molecules are attached to each other due to molecular bonding. When the temperature increases, the molecular bonding is getting weaker because the molecules are moving faster and trying to escape from each other. Therefore, the dynamic viscosity is decreasing.

For gas: Within the gas, the molecules are already free to move. When the temperature increases, there will be more vigorous molecular motion, more collisions and more momentum exchange, so the viscosity is increasing.

#### 3.2.2 Pressure

When the pressure increases, no matter gas or liquid, there will be more collisions and more momentum exchange, which will cause the increase of viscosity.

# References

[1] URL https://civilengineeronline.com/mech/stress-trans.php.