Jet Flame

1 Overview

Comparing with the non-reacting flow, the laminar jet flames have the following characteristics:

1. Fuel and oxidizer must react at some f
2. Heat release will cause the change in density, such as buoyancy effects
3. Must include diffusion of products
4. There will be source and sink terms in the species conservation equations

2 Flame Sheet Approximation

One important question we want to know is where will flame sit in terms of equivalence ratio. Based on the assumption of diffusion flame:

$$Da = \frac{\tau_{mix}}{\tau_{chem}} >> 1 \tag{1}$$

This means the chemistry is very fast, the reaction zone should be very thin because as soon as fuel and oxidizer meet, they will react.

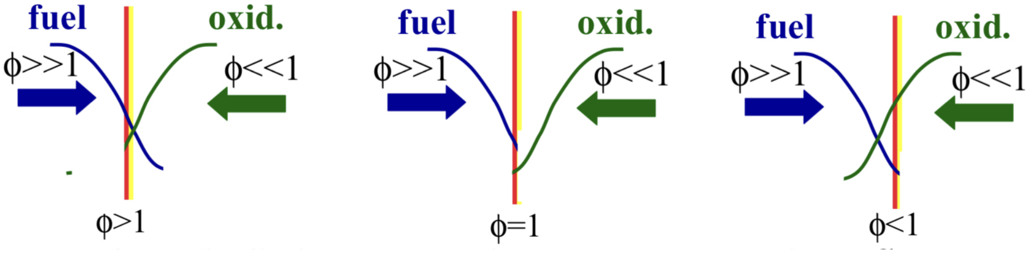


Figure 1: Flame Sheet Approximation

Therefore, we assume that **for fast, single-step reaction, flame sits at stoichiometric surface**, which is called **flame sheet approximation.**

Flame sheet approximation is not always valid. Classic failures include:

1. **Multistep Chemistry:** even with fast chemistry, will still have fuel and oxidizer coexisting

2. **Finite Rate Chemistry:** Solution will depend on chemical time scales and flow time scales

3 Jet Flame Descriptions

Based on the fuel-oxidizer ratio, we can divide the jet flame into two categories:

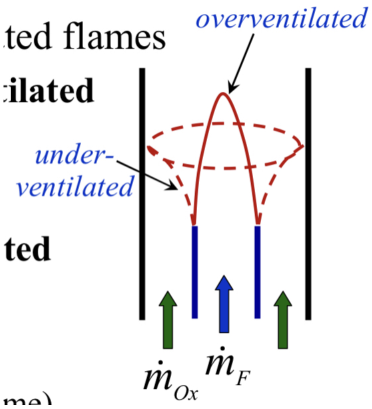


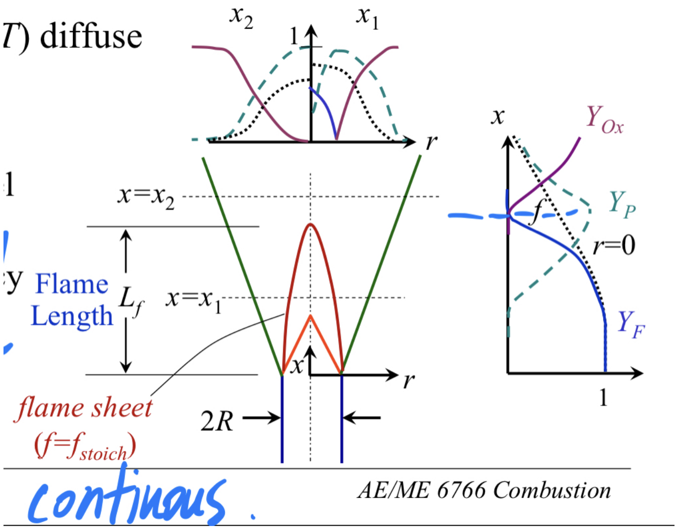
Figure 2: Ventilation

1. **Underventilated:** overall excess fuel (ventilation means air flow, "under" means less air):

$$\dot{m}_F/\dot{m}_{ox} > (\dot{m}_F/\dot{m}_{ox})_{stoich} \tag{2}$$

2. **Overventilated:** overall excess air:

$$\dot{m}_F/\dot{m}_{ox} < (\dot{m}_F/\dot{m}_{ox})_{stoich} \tag{3}$$



continuus.

Figure 3: Flame Description

Some remarks:

1. Fuel and oxidizer diffuse toward flame sheet
2. Products diffuse away from flame sheet
3. Flame tip position is affected by the buoyancy
4. **Non-premixed flame does not propagate**

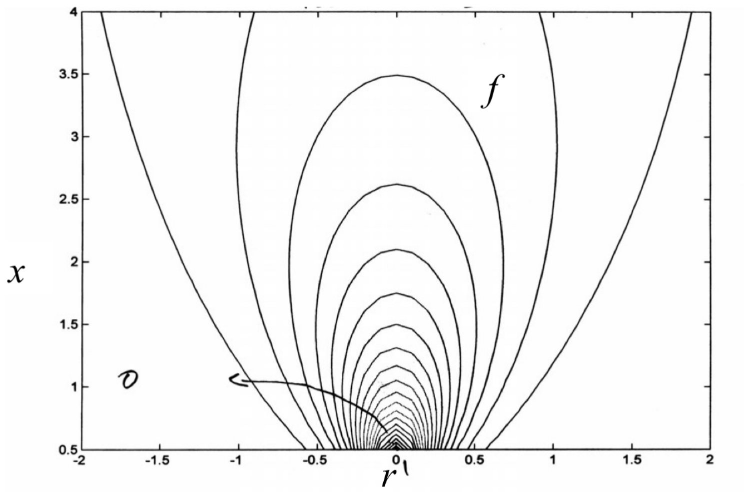


Figure 4: Mix Fraction Contour

If there is no heat release, then the flame shape will follow f_{stoich} contour, with $f = 1$ at the middle.

4 Governing Equations

4.1 Original Equations

Non-reacting jet assumptions:

1. Laminar
2. Steady
3. Axisymmetric
4. Quiescent/infinite reservoir
5. Fickian diffusion
6. No axial diffusion, but now not constant density

Flame assumptions:

1. Normal thermal diffusion
2. No radiative transfer

3. No viscous dissipation
4. Constant pressure
5. Flame sheet approximation
6. Le = 1

Mass Conservation:

$$\frac{\partial}{\partial x}(\rho u_x) + \frac{1}{r}\frac{\partial}{\partial r}(r\rho u_r) = 0 \tag{4}$$

Axial Momentum:

$$\underbrace{\frac{1}{r}\frac{\partial}{\partial x}(r\rho u_x u_x)}_{\text{axial conv.}} + \underbrace{\frac{1}{r}\frac{\partial}{\partial r}(r\rho u_r u_x)}_{\text{radial conv.}} - \underbrace{\frac{1}{r}\frac{\partial}{\partial r}(r\rho\nu\frac{\partial u_x}{\partial r})}_{\text{radial diffusion.}} = \underbrace{(\rho_\infty - \rho)g}_{\text{buoyancy, body force}} \tag{5}$$

Species Conservation:
Firstly, we know:

$$Y_P = 1 - Y_F - Y_{Ox} \tag{6}$$

We also have:

$$\underbrace{\frac{1}{r}\frac{\partial}{\partial x}(r\rho u_x Y_i)}_{\text{axial conv.}} + \underbrace{\frac{1}{r}\frac{\partial}{\partial r}(r\rho u_r Y_i)}_{\text{radial conv.}} - \underbrace{\frac{1}{r}\frac{\partial}{\partial r}(r\rho D\frac{\partial Y_i}{\partial r})}_{\text{radial diffusion.}} = \underbrace{\dot{m}_i'''}_{\text{source/sink}} \tag{7}$$

However, we have issues with boundary conditions. **Chemical term is 0 everywhere except at flame sheet**, so the flame is **boundary between inside and outside solutions**, but we do not know where the flame is. We know the mass fluxes into the flame governed by stochiometric proportions:

$$\sum_i v_i n_i \to \sum_j v_j n_j \tag{8}$$

$$\frac{\dot{m}_i'''}{\dot{m}_j'''} = \frac{v_i}{v_j}\frac{\bar{W}_i}{\bar{W}_j} \tag{9}$$

The energy equation also has the same issues.

4.2 Conserved Scalar

To solve this problem, we introduce **conserved scalars**, which are scalars that have no sources or sinks in the flow. In other words, these are scalar that "exists" on both sides of the flame (can change) and their integral is constant, just like the mass flow rate of jet fluid in non reacting case.

Several examples of conserved scalars:

1. **Total enthalpy:** $h_{sens} + h_{chem}$, if we ignore radiation, viscous dissipation and body force work

2. **Mixture fraction:** f
3. **Elemental Atom Mass Fraction:**

$$Z_i = \sum_{j=1}^{N} \mu_{ij} Y_j \tag{10}$$

Here μ_{ij} is the mass proportion of element i in species j. For example:

$$\mu_{H,CH4} = 4/16 \tag{11}$$

Using this method, the **species conservation** could be expressed as:

$$\frac{\partial}{\partial x}(r\rho u_x f) + \frac{\partial}{\partial r}(r\rho u_r f) - \frac{\partial}{\partial r}(r\rho D\frac{\partial f}{\partial r}) = 0 \tag{12}$$

Which means we only need one equation to determine composition due to flame sheet approximation(flame occurs at stochiometric equivalence ratio). For example, if $f > f_{stoich}$, there will be only fuel and products, so:

$$Y_F + Y_P = 1 \tag{13}$$

Similarly, the **energy conservation** could be expressed as (assume $Le = 1, \alpha = D$):

$$\frac{\partial}{\partial x}(r\rho u_x h) + \frac{\partial}{\partial r}(r\rho u_r h) - \frac{\partial}{\partial r}(r\rho D\frac{\partial h}{\partial r}) = 0 \tag{14}$$

Notice that if we apply unity Lewis number and Schmidt number assumption:

$$Le = \frac{\alpha}{D} = \frac{\nu}{D} = Sc = 1 \tag{15}$$

So:

$$\alpha = D = \nu \tag{16}$$

4.3 Normalization

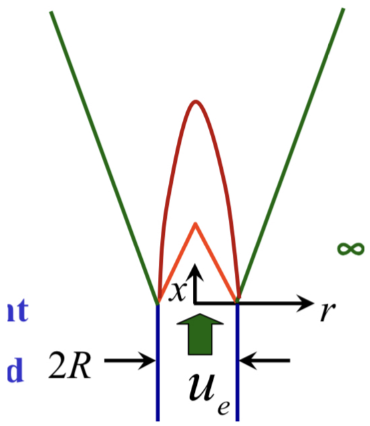


Figure 5: Normalization

$$x^* = x/R \tag{17}$$

$$r^* = r/R \tag{18}$$

$$u_x^* = u_x/u_e \tag{19}$$

$$u_r^* = u_r/u_e \tag{20}$$

$$\rho^* = \rho/\rho_e \tag{21}$$

$$h^* = \frac{h - h_{ox,\infty}}{h_{F,e} - h_{ox,\infty}} \tag{22}$$

Here, F, e means fuel, at the exit. Therefore, $h^* = 0$ in pure ambient, and $h^* = 1$ in pure jet fluid.

Now we define the **fraction of product mass that comes from "fuel"** as f_{stoich}, then:

$$f = Y_F + f_{stoich} Y_P \tag{23}$$

Which means the left fuel mass fraction plus the mass from fuel stored in product equal to the original fuel fraction from jet, which is the mixture fraction.

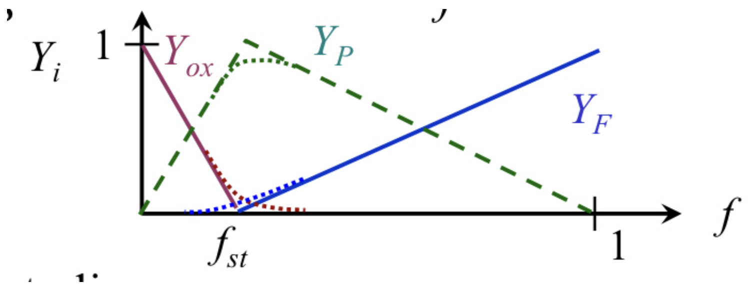


Figure 6: Mixture Fraction Relations

$\mathbf{f > f_{stoich}}$:

$$Y_{ox} = 0 \tag{24}$$

$$f = Y_F + f_{stoich} Y_P = Y_F + f_{stoich}(1 - Y_F) \tag{25}$$

$$Y_F = (f - f_{stoich})/(1 - f_{stoich}) \tag{26}$$

$$Y_P = (1 - f)/(1 - f_{stoich}) \tag{27}$$

$\mathbf{f < f_{stoich}}$:

$$Y_F = 0 \tag{28}$$

$$Y_P = f/f_{stoich} \tag{29}$$

$$Y_{ox} = 1 - Y_P = 1 - f/f_{stoich} \tag{30}$$

$\mathbf{f = f_{stoich}}$:

$$Y_P = 1 \tag{31}$$

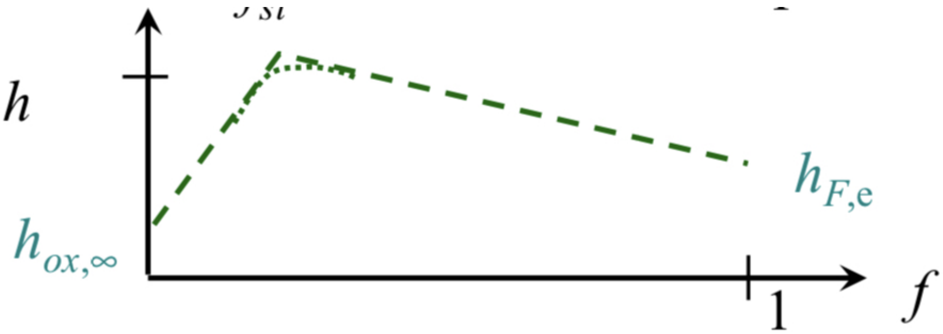


Figure 7: Enthalpy Relations

From the definition of h^*, we know $h^* = f$. Therefore:

$$h = h_{ox,\infty} + f(h_{F,e} - h_{ox,\infty})x \tag{32}$$

Recall that:

$$\alpha = D = \nu \tag{33}$$

So we can use single parameter η to express u_x^*, f, h^*. The boundary conditions include:

1. $@r^* = 0, x : \partial\eta/\partial r = 0, u_r^* = 0$
2. $@r^* = \infty : \eta = 0$
3. $@x^* = 0, r^* > 1 : \eta = 0$
4. $@x^* = 0, r^* \leq 1 : \eta = 1$

After normalization, **Mass Conservation:**

$$\frac{\partial}{\partial x^*}(\rho^* u_x^*) + \frac{1}{r^*}\frac{\partial}{\partial r^*}(r^* \rho^* u_r^*) = 0 \tag{34}$$

Momentum Conservation:

$$\frac{\partial}{\partial x^*}(r^* \rho^* u_x^* \eta) + \frac{\partial}{\partial r^*}(r^* \rho^* u_r^* \eta) - \frac{\partial}{\partial r^*}(r^* \frac{1}{Re_j}\frac{\partial \eta}{\partial r^*}) = r^* \frac{gR}{u_e^2}(\rho_\infty^* - \rho^*) \tag{35}$$

Here,

$$\frac{1}{Fr} = \frac{gR}{u_e^2} \tag{36}$$

The Froude number (Fr) is a dimensionless number used in fluid dynamics and hydrodynamics, comparing the flow inertia to the external field (often gravitational).

Species Conservation:

$$\frac{\partial}{\partial x^*}(r^* \rho^* u_x^* \eta) + \frac{\partial}{\partial r^*}(r^* \rho^* u_r^* \eta) - \frac{\partial}{\partial r^*}(r^* \frac{1}{Re_j Sc} \frac{\partial \eta}{\partial r^*}) = 0 \tag{37}$$

4.4 Solution

Recall that:

$$\alpha = D = \nu \tag{38}$$

Also assume constant density, then assume constant density ($\rho^* = 1$), then all conservation equations become:

$$\frac{\partial}{\partial x^*}(r^* u_x^* \eta) + \frac{\partial}{\partial r^*}(r^* u_r^* \eta) - \frac{\partial}{\partial r^*}(r^* \frac{1}{Re_j} \frac{\partial \eta}{\partial r^*}) = 0 \tag{39}$$

This will give identical solution as for nonreacting jet:

$$\frac{u_x}{u_e} \frac{x}{R} \frac{1}{Re_j} = f \frac{x}{R} \frac{1}{Re_j} = h^* \frac{x}{R} \frac{1}{Re_j} = \frac{3}{8}[1 + \frac{\zeta^2}{4}]^{-2} \tag{40}$$

Recall that:

$$\zeta = \frac{\sqrt{3}}{4} \frac{r}{R} Re_j (\frac{R}{x}) \tag{41}$$

So the final solution is:

$$\frac{3/8}{(1 + (3/64) Re_j^2 (r^2/x^2))^2} \tag{42}$$

Recall the flame sheet approximation, flame location will be at $f = f_{stoich}$. Normally, we have diluted oxygen and pure fuel coming from the jet. Then:

$$f_{stoich} = \frac{1}{1 + \frac{(O/F)_{stoich}}{Y_{Ox,\infty}}} \tag{43}$$

Here $(O/F)_{stoich}$ is the stoichmetric oxygen/fuel mass ratio.

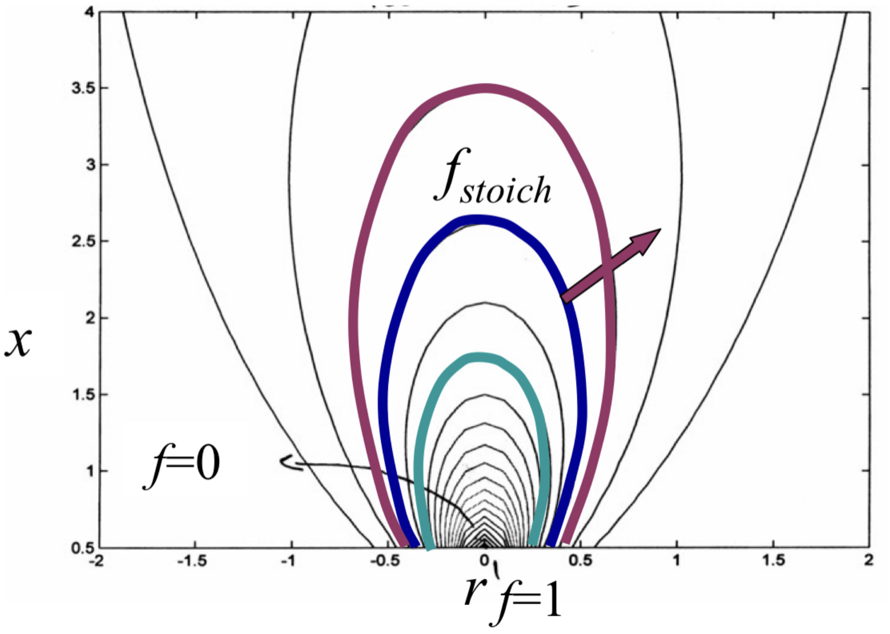


Figure 8: Flame Shape Contour

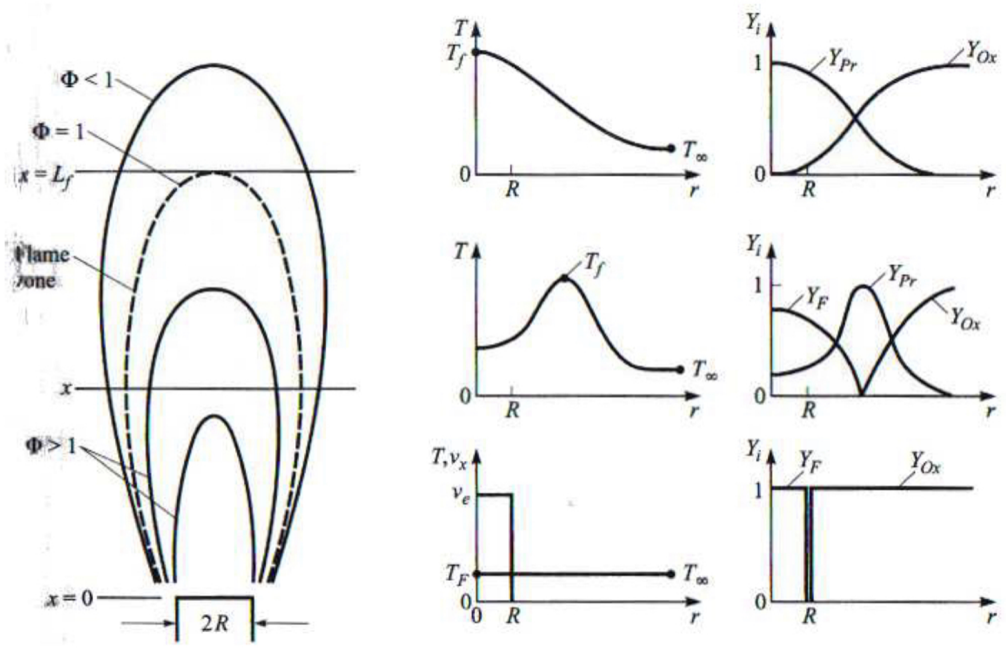


Figure 9: Flame Structure

If we dilute oxidizer, then the mass fraction of oxidizer in the environment $(Y_{ox,\infty})$ decrease, f_{stoich} will also decrease. From the contour we know the flame **gets bigger.**

If we dilute fuel, we need more fuel to burn, so $(O/F)_{stoich}$ will decrease, so f_{stoich} will increase, flame **gets smaller.**

We are also interested in flame length for this type of flame. Recall that:

$$f\frac{x}{R}\frac{1}{Re_j} = \frac{3/8}{(1+(3/64)Re_j^2(r^2/x^2))^2} \tag{44}$$

Rearrange this equation we get:

$$\frac{r_{flame}(f=f_{stoich})}{x} = \frac{8}{\sqrt{3}}\frac{1}{Re_j}[(\frac{3}{8}\frac{Re_j}{f_{stoich}}\frac{R}{x})^{1/2}-1]^{1/2} \tag{45}$$

When radial distance $r_{flame} = 0$, we can get the flame length, which is defined as the distance from the base of a flame to the tip of the flame, where combustion ceases or becomes too small to observe.

$$(\frac{3}{8}\frac{Re_j}{f_{stoich}}\frac{R}{x})^{1/2}-1=0 \tag{46}$$

$$x_{flame} = L_f = \frac{3}{8}Re_j\frac{R}{f_{stoich}} = \frac{3}{8}\frac{u_eR}{\nu}\frac{R}{f_{stoich}} \tag{47}$$

Recall the definition of volumetric flowrate:

$$Q_e = \frac{\dot{m}_e}{\rho_e} = \frac{\rho_e u_e A}{\rho_e} = u_e A = u_e \pi R^2 \tag{48}$$

Therefore:

$$L_f = \frac{3}{8\pi}\frac{1}{f_{stoich}}\frac{Q_e}{\nu} \tag{49}$$

Again, based on the assumption:

$$\alpha = D = \nu \tag{50}$$

Therefore:

$$L_f \propto \frac{Q_e/D}{f_{stoich}} \tag{51}$$

So we can increase the flame length by increasing Q_e/D or decrease f_{stoich}, for example diluting oxidizer (discussed before).

In the other way,

$$L_f = \frac{3}{8\pi}\frac{1}{f_{stoich}}\frac{Q_e}{\nu} = \frac{3}{8\pi}\frac{1}{f_{stoich}}\frac{\dot{m}_j/\rho_e}{\mu/\rho_e} = \frac{3}{8\pi}\frac{1}{f_{stoich}}\frac{\dot{m}_j}{\mu} \tag{52}$$

Which means that flame length is not a function of pressure. We can use this principle to reduce combustor length by having distributed nozzle.

5 Partially Premixed Combustion

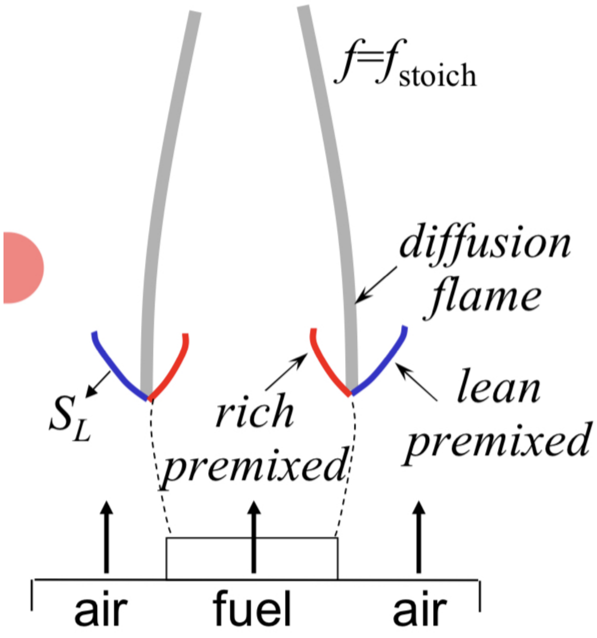


Figure 10: Partially Premixed Combustion

Practical systems involve combination of nonpremixed and premixed combustion, including **partially premixed combustion.**

The graph above shows a classical triple point flame. The premixed flames anchor diffusion flames.

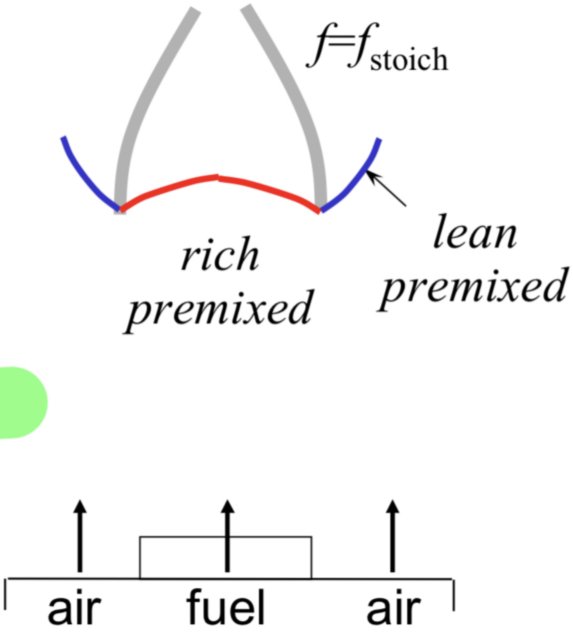


Figure 11: Higher Speed Partially Premixed Combustion

With higher jet flow rate, there will be longer distance to ignition point, which means less diffusion flame and more premixed flame. The flame keeps lifting off, if we keep increasing jet velocity, we will reach a point even stoichiometric flame too slow to propagate, so the flame will **blow off**.