Jet Mixing

1 Nonpremixed Combustion

Non-premixed combustion, also known as diffusion combustion, is a type of combustion process where the fuel and oxidizer (often air) are not mixed before introduction to the combustion zone. Instead, combustion takes place when the fuel and oxidizer come into contact with each other in the combustion chamber.

For the premixed combustion, we only need to consider **kinetics and reactant-product diffusion**, now we also need to consider fuel-oxidizer mixing. To simplify the problem, we use laminar flow, only consider **diffusional mixing**. We also assume fast kinetics, then:

$$Da = \tau_{mix}/\tau_{chem} \rightarrow \infty \tag{1}$$

Therefore burning rate is limited by diffusion, we usually call it **diffusional flames**.

2 Jet Mixing

2.1 Overview

Before considering laminar jet flames, we first need to study the **laminar, nonreacting jets**. Some characteristics:

1. Fuel jet issuing into infinite reservoir of quiescent oxidizer
2. High velocity of jet will decrease as jet expands and is slowed down by viscous interaction with surrounding fluid
3. High concentration of fuel in jet will decrease as jet fluid mixes with low concentration surrounding fluid

2.2 Governing Equation

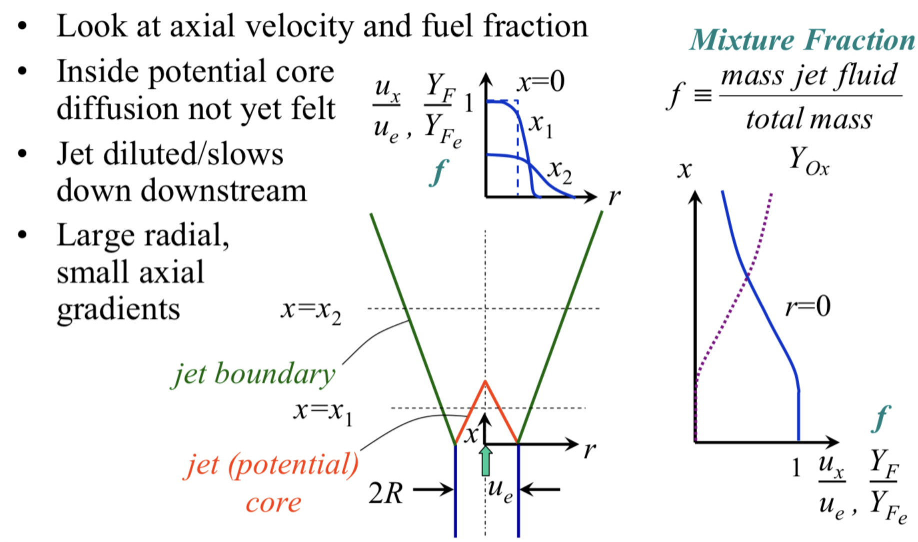

Figure 1: Jet Mixing

Mixture Fraction:

$$f = \frac{\text{mass from jet fluid}}{\text{total mass}} \tag{2}$$

Assumptions:

1. Quiescent surroundings
2. Steady
3. Nonreacting
4. Laminar
5. Axisymmetric flow

Mass Conservation:

$$\frac{\partial u_x}{\partial x} + \frac{1}{r}\frac{\partial (r u_r)}{\partial r} = 0 \tag{3}$$

Axial Momentum:

$$u_x \frac{\partial u_x}{\partial x} + u_r \frac{\partial u_x}{\partial r} = \frac{\nu}{r}\frac{\partial}{\partial r}(r \frac{\partial u_x}{\partial r}) \tag{4}$$

Notice that here we ignore axial diffusion, so this is not valid for $x < R$, where there is large axial diffusion.

Species:

$$u_x \frac{\partial f}{\partial x} + u_r \frac{\partial f}{\partial x} = \frac{D}{r}\frac{\partial}{\partial r}(r \frac{\partial f}{\partial r}) \tag{5}$$

If we assume **unity Schmidt number**:

$$Sc = \frac{\nu}{D} = 1 \tag{6}$$

Then the species equation and the axial momentum equation are the same.
Boundary Conditions:

1. $@x = 0, r \leq R, u_x/u_e = f = 1$
2. $@x = 0, r > R, u_x/u_e = f = 0$
3. $@r = \infty, x, u_x/u_e = f = 0$
4. $@r = 0, x, \frac{\partial(u_x/u_e)}{\partial r} = \frac{\partial f}{\partial r} = 0; u_r = 0$

There are two conserved quantities along any plane normal to jet axis:
Total axial momentum:

$$J_e = 2\pi \int_0^\infty \rho u_x u_x r dr = \rho_e u_e^2 \pi R^2 \tag{7}$$

Total jet fluid:

$$\dot{m}_e = 2\pi \int_0^\infty \rho u_x f r dr = \rho_e u_e \pi R^2 \tag{8}$$

Using self-similarity method to solve these equations (not shown here), we can get the final solutions. Assume an intermediate term η:

$$\eta = (\frac{3\rho_e J_e}{16\pi})^{1/2} \frac{1}{\mu} \frac{r}{x} = \frac{\sqrt{3}}{4} \frac{r}{R} Re_j (\frac{x}{R})^{-1} \tag{9}$$

Here Re_j is the jet exit Reynolds number:

$$Re_j = \frac{u_e R}{\nu} \tag{10}$$

The axial velocity solution is:

$$\frac{u_x}{u_e} = \frac{3}{8\pi} \frac{J_e/u_e}{\mu x} [1 + \frac{\eta^2}{4}]^{-2} = \frac{3}{8} Re_j (\frac{x}{R})^{-1} [1 + \frac{\eta^2}{4}]^{-2} \tag{11}$$

Mixture fraction is:

$$f = \frac{3}{8\pi} \frac{Q_e}{Dx} 1 + \frac{\eta^2}{4}]^{-2} = \frac{3}{8} Re_j (\frac{x}{R})^{-1} [1 + \frac{\eta^2}{4}]^{-2} \tag{12}$$

Here, Q_e is the volumetric flowrate:

$$Q_e = \frac{\dot{m}_e}{\rho_e} \tag{13}$$

Radial velocity solution is:

$$\frac{u_r}{u_e} = \frac{\sqrt{3}}{16} (\frac{x}{R})^{-1} (\eta - \frac{\eta^3}{4}) [1 + \frac{\eta^2}{4}]^{-2} \tag{14}$$

2.3 Dependence

In axial direction, based on the equations before:

$$\frac{u_x}{u_e}, f(r=0) \propto Re_j(\frac{x}{R})^{-1} \tag{15}$$

Therefore:

1. Decay as $1/x$
2. If Re is high, the decay rate will be slower

In radial direction,

$$\frac{u_x(r)}{u_x(r=0)} = \frac{f(r)}{f(r=0)} = [1+\frac{\eta^2}{4}]^{-2} \propto \frac{1}{[Re_j^2 r^2 + const]^2} \tag{16}$$

Therefore **jet will be thinner at high Re**, because the flow has higher axial speed, but the diffusive velocity is the same.

If $Sc \neq 1$:

$$\frac{f(r)}{f(r=0)} = [\frac{u_x(r)}{u_x(r=0)}]^{Sc} \tag{17}$$

If $Sc < 1$, concentration profiles will be broader than velocity profile.

2.4 Jet Spread Rate

We define the **jet half-width** $r_{1/2}$ as the radial distance when axial velocity, mixture fraction drop to half of centerline value for given x.

$$\frac{1}{2} = \frac{u_x(r)}{u_x(r=0)} = \frac{f(r)}{f(r=0)} = [1+\frac{\eta^2}{4}]^{-2} \tag{18}$$

We get:

$$\eta \approx 1.287 \tag{19}$$

Recall:

$$\eta = \frac{\sqrt{3}}{4}\frac{r}{R}Re_j(\frac{R}{x}) \tag{20}$$

Therefore:

$$\frac{r_{1/2}}{x} = \frac{4}{\sqrt{3}}\frac{1}{Re_j}\eta \approx 2.97 Re_j^{-1} \tag{21}$$

Then we define the **jet half angle** as:

$$\alpha = tan^{-1}(\frac{r_{1/2}}{x}) = tan^{-1}(2.97 Re_j^{-1}) \tag{22}$$

2.5 Jet Entrainment

We are also interested in total mass crossing any downstream plane. Recall the mass flow rate expression:

$$\dot{m} = 2\pi \int_0^\infty \rho u_x r dr \tag{23}$$

Recall the axial velocity solution:

$$\frac{u_x}{u_e} = \frac{3}{8\pi} \frac{J_e/u_e}{\mu x} [1 + \frac{\eta^2}{4}]^{-2} \tag{24}$$

Then:

$$\dot{m} = 2\pi \int_0^\infty \frac{3}{8\pi} \frac{J_e}{\mu x} [1 + \frac{\eta^2}{4}]^{-2} r dr \tag{25}$$

Recall that:

$$\eta = (\frac{3\rho_e J_e}{16\pi})^{1/2} \frac{1}{\mu} \frac{r}{x} \tag{26}$$

Then:

$$r = \frac{4}{\sqrt{3}} (\frac{\pi}{\rho J_e})^{1/2} \mu x \eta \tag{27}$$

$$r dr = \frac{16}{3} \frac{\pi}{\rho J_e} (\mu x)^2 \eta d\eta \tag{28}$$

Therefore:

$$\dot{m} = 4\pi \mu x \int_0^\infty \frac{\eta d\eta}{(1 + \eta^2/4)^2} \tag{29}$$

Assume $y = \eta^2/4$,

$$\dot{m} = 8\pi \mu x \int_0^\infty \frac{dy}{(1 + y)^2} = 8\pi \mu x \tag{30}$$

Therefore, total mass flow rate increases downstream.
However:

$$\frac{d\dot{m}}{dx} = 8\pi \mu = const \tag{31}$$

The flow still has constant entrainment rate.