Thermodynamic Cycle

1 Overview

A thermodynamic cycle refers to a sequence of processes that begin and end at the same thermodynamic state. It involves changes in temperature, pressure, and energy, and these processes will often involve the transfer of heat and work between the system undergoing the cycle and its surroundings.

2 Energy Conservation

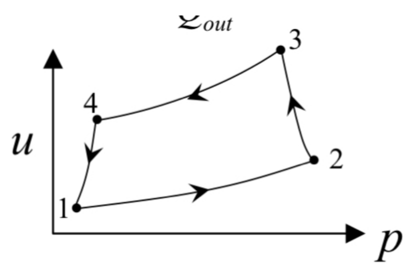

Figure 1: Cycle Example

For a closed system operating in steady state, recall the first law:

$$dU = \delta Q_{net,in} + \delta W_{net,in} = \delta Q_{net,in} - \delta W_{net,out} \tag{1}$$

Based on the definition of cycle, at the end it will return back to initial state. Therefore:

$$\delta Q_{net,in} = \delta W_{net,out} \tag{2}$$

In rate version:

$$\dot{Q}_{net,in} = \dot{Q}_{in} - \dot{Q}_{out} = -\dot{W}_{net,in} = -(\dot{W}_{in} - \dot{W}_{out}) = \dot{W}_{net,out} \tag{3}$$

3 Cycle Directions

3.1 Heat Engine

For a heat engine, we want to produce work/power, such as IC engines.

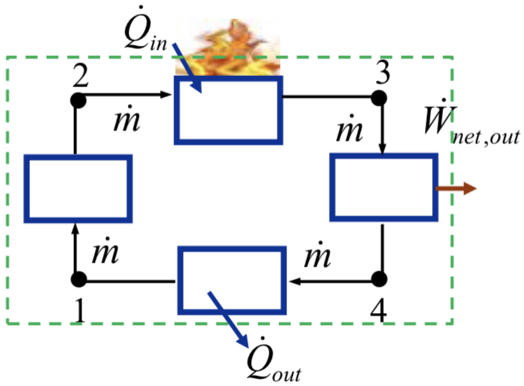

Figure 2: Heat Engine

We can define **cycle (thermal) efficiency**:

$$\eta = \frac{W_{net,out}}{Q_{in}} = \frac{\dot{W}_{net,out}}{\dot{Q}_{in}} \tag{4}$$

Only some of the energy/heat transferred to the working fluid is converted to output power. **The rest is wasted to raise energy/temperature of the colder environment**

3.2 Thermal Transfer System

Thermal transfer system move "heat" from colder to hotter system, such as refrigerators, air-conditioners.

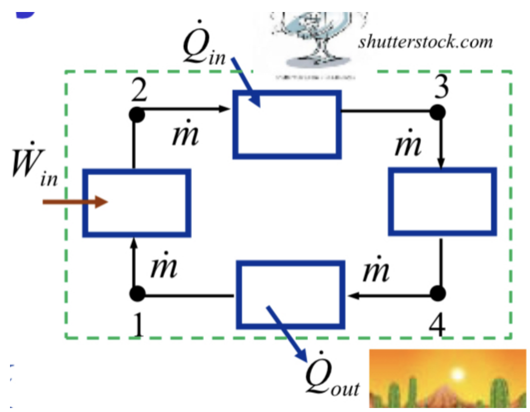

Figure 3: Thermal Transfer System

Within this cycle:

$$\dot{Q}_{net,out} = \dot{W}_{in} \tag{5}$$

In terms of the energy removed by heat transfer from the cold system:

$$\dot{Q}_{in,cold} = \dot{Q}_{out,hot} - \dot{W}_{in} \tag{6}$$

Therefore if we have to supply work to run this system, we will **raise the energy/temperature of the hot surroundings more than we cool the target object**.

4 Process Summary

1. **Isentropic:** $dS = 0$, no entropy change
2. **Isothermal:** $dT = 0$, no temperature change
3. **Adiabatic:** No heat transfer
4. **Isobaric:** Constant pressure
5. **Isochoric:** Constant volume

5 Detailed Cycle Analysis

5.1 Carnot Cycle

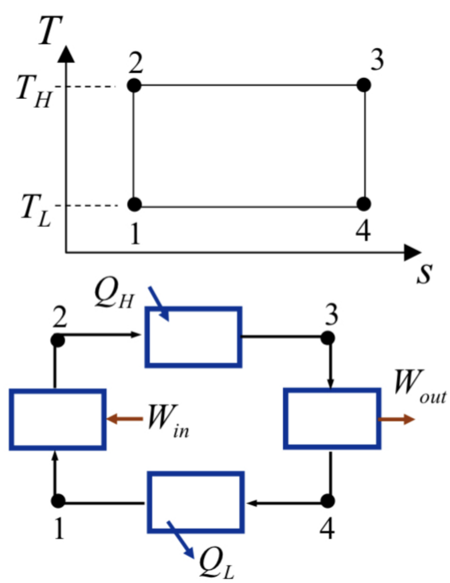

Figure 4: Carnot Cycle

The Carnot cycle is a theoretical thermodynamic cycle that demonstrates the maximum possible efficiency a heat engine can achieve when converting heat into work. It is a completely reversible cycle.

The processes include:

1. $1 \rightarrow 2$: **Isentropic (Adiabatic) Compression**, with work but no heat transfer, the compression will cause the increase of temperature

2. 2 → 3: **Isothermal Expansion**, absorb heat from Q_H and then the gas expands, but keeps the temperature the same.

3. 3 → 4: **Isentropic (Aiabatic Expansion)**, the substance expands without exchanging heat with the surroundings (an adiabatic process). During this phase, the temperature of the substance drops because the substance is doing work on the piston by expanding, but no heat is being added.

4. 4 → 1: **Isothermal Compression**, the working substance releases heat to a cold reservoir while the piston compresses the gas. This is also a reversible process.

Recall the definition of the cycle efficiency:

$$\eta = \frac{W_{net,out}}{Q_{in}} \tag{7}$$

In carnot cycle case:

$$\eta = \frac{W_{net,out}}{Q_H} \tag{8}$$

Based on energy conservation:

$$W_{net,out} = W_{out} - W_{in} = Q_H - Q_L \tag{9}$$

So:

$$\eta = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H} \tag{10}$$

Because in Carnot Cycle we assume all the processes are reversible, so:

$$\eta = \frac{Q}{T} \tag{11}$$

$$\eta = 1 - \frac{T_L(s_4 - s_1)}{T_H(s_3 - s_2)} \tag{12}$$

From the T-S diagram, we know $s_3 = s_4$, $s_1 = s_2$, therefore:

$$\eta_{Carnot} = 1 - \frac{T_L}{T_H} \tag{13}$$

Some remarks:

1. To maximize η, we need $T_H >> T_L$, so we need to add heat a high T as possible and reject heat at low T as possible.

2. **Even though every process in Carnot Cycle is reversible, we still could not get 100% efficiency unless $T_L = 0$**

3. **Carnot Cycle has the highest efficiency comparing with other cycles because all of its processes are reversible, proven before.**

5.2 Otto Cycle

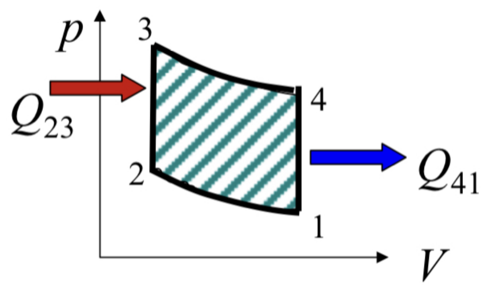

Figure 5: Otto Cycle

Processes:

1. $1 \rightarrow 2$: Compression (adiabatic)
2. $2 \rightarrow 3$: Heat transfer into fluid from combustion (no work)
3. $3 \rightarrow 4$: Expansion (adiabatic)
4. $4 \rightarrow 1$: Exhaust and heat transfer out, no net work

The shaded area represents **net heat transfer in** $Q_{23} - Q_{41}$ and **net work done by system** $W_{34} - W_{12}$.