First Law

1 Closed System First Law

1.1 Equations

Energy cannot be created or destroyed in an isolated system; it can only change forms or be transferred between systems.

In equation:

$$dE = \delta Q + \delta W \tag{1}$$

Some remarks:

1. dE is the change in the energy. Because E is a state function, exists whether system is changing or not, could differentiate, so we use d.

2. δW is the infinitesimal amount of work, here we define $\delta W > 0$ for **work done to CM**, like absorbing energy.

3. δQ is the infinitesimal amount of heat transfer, $\delta Q > 0$ for **heat transfer into CM**.

4. Q and W are path functions, not differentiable function of our CM, so we use δ.

First Law has many other versions. For example, in mass basis:

$$de = \delta q + \delta w \tag{2}$$

Or in rates:

$$\frac{dE}{dt} = \dot{Q} + \dot{W} \tag{3}$$

Or on an integral basis:

$$\Delta E_{12} = Q_{12} + W_{12} \tag{4}$$

1.2 Applications

If only compression/expansion work possible, then we define:

$$\delta W = -pdV \tag{5}$$

Because when system's volume is decreasing, it is compressed, the environment is doing work to the system, so $\delta W > 0$.

If the volume is constant, and we ignore the **kinetic energy and potential energy**, we get the following equations:

$$\cancelto{dU}{dE} = \delta Q - \cancelto{0}{pdV} \tag{6}$$

$$dU = \delta Q \tag{7}$$

Recall that when volume is constant, we also have:

$$du = c_v dT + \frac{\partial u}{\partial v}|_T \cancelto{0}{dv} \tag{8}$$

Therefore:

$$\delta Q = mc_v dT \tag{9}$$

In finite change:

$$Q_{12} = m \int_{T_1}^{T_2} c_v dT \tag{10}$$

Similarly, **if the pressure is constant**, we get:

$$dU = \delta Q - pdV \tag{11}$$

$$\delta Q = dU + pdV = dU + (pdV + Vdp) = dU + d(pV) \tag{12}$$

Based on the definition of enthalpy:

$$H = U + pV \tag{13}$$

Therefore:

$$\delta Q = dH = mc_p dT \tag{14}$$

2 Open System First Law

2.1 Reynolds Transport Theorem

To convert a control mass law into a control volume law, we can track the fluxes of the property that is conserved. This method is called **Reynolds Transport Theorem**:

$$\frac{dB}{dt}|_{CM} = \frac{d}{dt}\int_{CV} \rho\beta dV + \int_{CS} \rho\beta(\underline{\boldsymbol{u}} \cdot \underline{\hat{\boldsymbol{n}}})dA \tag{15}$$

Some remarks:

1. $\frac{dB}{dt}$ is the expression from control mass conservation law.
2. B is an extensive property, with intensive version β

This will lead to a **PICO** relationship:

$$\text{Production} + \text{Input} = \text{Change in time} + \text{Output} \tag{16}$$

Take mass conservation as an example:

$$B = m, \beta = 1, \frac{dm}{dt}|_{CM} = 0 \tag{17}$$

Therefore:

$$0 = \frac{d}{dt}\int_{CV} \rho dV + \int_{CS} \rho(\underline{\boldsymbol{u}} \cdot \underline{\hat{\boldsymbol{n}}})dA \tag{18}$$

Which can be expressed as:

$$\text{Production} = \text{Change in time} + \text{Output} - \text{Input} \tag{19}$$

2.2 Derivation

Now we come back to open system first law. Here:

$$B = E, \beta = e \tag{20}$$

$$\frac{dE}{dt}|_{CM} = \dot{Q} + \dot{W} \tag{21}$$

The work in general include:

$$\dot{W} = \dot{W}_u + \dot{W}_{boundary} + \dot{W}_{body} \tag{22}$$

Here:

1. $\dot{W}_u$ represents shaft work. **Shaft work** refers to the mechanical work that is either done by the system on its surroundings or by the surroundings on the system. Typically, it represents the work associated with rotating equipment, such as turbines, compressors, and pumps.
2. $\dot{W}_{boundary}$ represents flow work if we assume simple compressible substances. The entering or leaving of mass has to push way into the system, same as pdV. Therefore:

$$\dot{W}_{boundary} = -\int_{CS} p(\underline{\boldsymbol{u}} \cdot \underline{\hat{\boldsymbol{n}}})dA \tag{23}$$

3. $\dot{W}_{body}$ refers to the work done by body force.

Therefore, the master equation of open system first law is:

$$\dot{Q} + \dot{W}_u + \dot{W}_{boundary} + \dot{W}_{body} = \frac{d}{dt}\int_{CV} \rho e dV + \int_{CS} \rho e(\underline{\boldsymbol{u}} \cdot \underline{\hat{\boldsymbol{n}}})dA \tag{24}$$

Where the LHS is the **energy transfer rate** into CV, and RHS are **rate of change of energy inside CV** and **net rate energy flows out of CV carried by mass**.

If we ignore viscous force and body force, we have:

$$\begin{aligned}\int_{CS} \rho e(\underline{\boldsymbol{u}} \cdot \underline{\hat{\boldsymbol{n}}})dA - \dot{W}_{boundary} &= \int_{CS} \rho e(\underline{\boldsymbol{u}} \cdot \underline{\hat{\boldsymbol{n}}})dA + \int_{CS} p(\underline{\boldsymbol{u}} \cdot \underline{\hat{\boldsymbol{n}}})dA \\ &= \int_{CS} (e + \frac{P}{\rho})(\rho\underline{\boldsymbol{u}} \cdot \underline{\hat{\boldsymbol{n}}})dA \\ &= \int_{CS} (u + \frac{v^2}{2} + \frac{P}{\rho})(\rho\underline{\boldsymbol{u}} \cdot \underline{\hat{\boldsymbol{n}}})dA \\ &= \int_{CS} (h + \frac{v^2}{2})(\rho\underline{\boldsymbol{u}} \cdot \underline{\hat{\boldsymbol{n}}})dA\end{aligned} \tag{25}$$

Recall that:

$$H = pV \tag{26}$$

Then:

$$h = pv = \frac{p}{\rho} \tag{27}$$

If we further assume steady state ($d/dt = 0$), we get:

$$\dot{Q} + \dot{W}_u = \int_{CS} (h + \frac{v^2}{2})(\rho\underline{\boldsymbol{u}} \cdot \underline{\hat{\boldsymbol{n}}})dA \tag{28}$$

Recall that the mass flow rate could be expressed as:

$$\dot{m} = \rho u A \tag{29}$$

Therefore:

$$\dot{Q} + \dot{W}_u = \dot{m}_{out}(h_{out} + \frac{v_{out}^2}{2}) - \dot{m}_{in}(h_{in} + \frac{v_{in}^2}{2}) \tag{30}$$

If we have constant mass flow rate, and ignore kinetic energy:

$$\dot{Q} + \dot{W}_u = \dot{m}(h_{out} - h_{in}) \tag{31}$$

$$q + w_u = h_{out} - h_{in} \tag{32}$$