

Specific Heat

1 Derivation

In general, for the simple compressible substances, the only reversible work mode is compression/expansion, so we only need 2 independent intensive variables to get the rest of the intensive variables. Therefore, we can relate the internal energy with 2 intensive variables:

$$u = u(T, v) \quad (1)$$

Using multivariable calculus, we can write:

$$du = \left. \frac{\partial u}{\partial T} \right|_v dT + \left. \frac{\partial u}{\partial v} \right|_T dv \quad (2)$$

If we assume constant volume, then we have:

$$du = \left. \frac{\partial u}{\partial T} \right|_v dT \quad (3)$$

So we define the specific heat at constant volume as:

$$c_v = \left. \frac{\partial u}{\partial T} \right|_v \quad (4)$$

The unit of c_v is the same as R, which is $J/kg \cdot K$.

2 Perfect Gas Assumptions

2.1 Thermally Perfect Gas (TPG)

TPG assumptions include:

1. $\left. \frac{\partial u}{\partial v} \right|_T = 0, du = c_v dT$
2. u is function of only T : $u = u(T)$
3. Specific heat is function of only T : $c_v = c_v(T)$

Some remarks:

1. **Energy is "relative" in TD ($u_2 - u_1$), no absolute energy**

2. u , c_v and T are not independent

As the temperature of a gas increases, the energy can start to excite rotational and vibrational modes of the molecules, and even ionize the gas. All these processes effectively increase the heat capacity of the gas, which wouldn't be captured with a simple ideal gas model. Therefore, normally the specific heat increase with the increase of temperature.

Therefore, we can express the internal energy as:

$$u_2 - u_1 = \int_{T_1}^{T_2} c_v dT \quad (5)$$

2.2 Carlhorically Perfect Gas (CPG)

At the base of TPG, if we further assume c_v is a constant, we get CPG. Therefore:

$$u_2 - u_1 = \int_{T_1}^{T_2} c_v dT = c_v(T_2 - T_1) \quad (6)$$

When the temperature is high, the vibrational motions of molecules become important so CPG assumptions may not valid anymore.

3 Specific Heat Ratio

In thermodynamics, **enthalpy** is a measure of the total energy of a thermodynamic system. It includes both the internal energy, which is the energy required to create a system, and the amount of energy required to make room for it by displacing its environment and establishing its volume and pressure.

$$h = u + pv \quad (7)$$

Taking derivative, we can get:

$$dh = d(u + pv) = du + d(pv) \quad (8)$$

Recall the ideal gas assumption:

$$dh = du + d(RT) = c_v dT + R dT = (c_v + R) dT \quad (9)$$

Here, we define the **specific heat at constant pressure** as:

$$c_p = c_v + R \quad (10)$$

Therefore, for TPG:

$$dh = c_p dT, c_p(T) = c_v(T) + R \quad (11)$$

We also define the **specific heat ratio** as:

$$\gamma = \frac{c_p}{c_v} \quad (12)$$

Rearrange we can also get:

$$\frac{c_p}{R} = \frac{\gamma}{\gamma - 1} \quad (13)$$

$$\frac{c_v}{R} = \frac{1}{\gamma - 1} \quad (14)$$

Normally, $\gamma = 1.4$ for diatomic gases at low temperature, without activation of vibrational energy mode.