Markov Chain

1 Introduction

A Markov Chain is a mathematical model that describes a sequence of possible events in which the probability of each event **depends only on the state attained in the previous event**. It is a conceptual model for discrete states and time, and **stochastic** (outcomes based upon random probability) evolution.

2 Elements

2.1 States

These are the possible situations or conditions the system can be in. Each state is mutually exclusive, meaning the system can only be in one state at a given time. Notice that the state of the system include **states of all nodes**.

2.2 Transitions

These represent the movement or changes between states. Each transition has an associated probability.

2.3 Transition Probabilities

These are the probabilities that the system moves from one state to another during a transition. They form the core of a Markov Chain. The probabilities associated with all possible transitions from a particular state should sum to 1.

2.4 Initial Distribution

This describes the probability distribution of the system's state at the start (or time t=0). It indicates the likelihood that the system begins in any particular state.

3 Case Study

Assume we want to model the spread of an infectious disease.

3.1 Case Setup

We have the following parameters:

1. k: Duration of infection. In this case, k = 2.
2. τ: Infection probability
3. Node label 0: Susceptible
4. Node label -1: Recovered
5. Node label 1 and 2: Infection date 1 and 2.

$$(b_1, b_2, b_3) = (2, 0, -1)$$

Figure 1: State Setup.

3.2 Transition

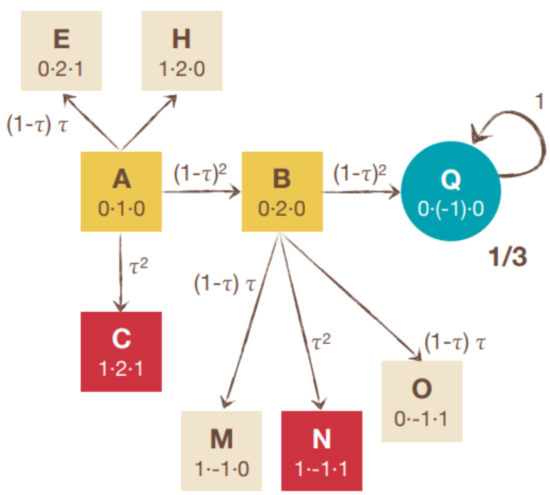
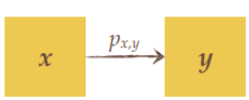


Figure 2: State Transition.

We define the transition from x to y as:



$$p_{x,y} \equiv \Pr[Y|X]$$

Figure 3: Transition from X to Y.

Then we could have the transition matrix:

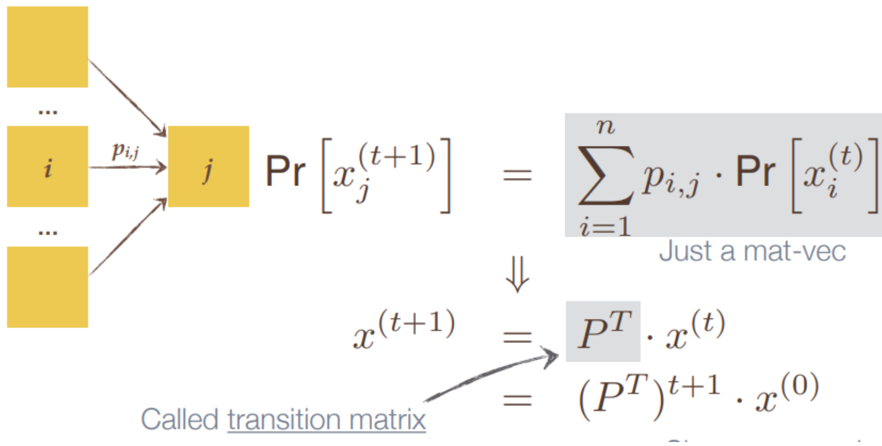


Figure 4: Transition Matrix.

The total number of the states could be expressed as:

$$(k+2)^n \tag{1}$$

Where:

1. n: the number of patients
2. 2: the number of safe states, which is susceptible and recovered.